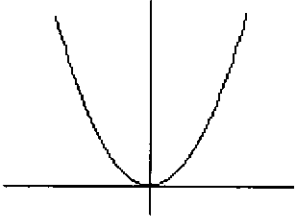
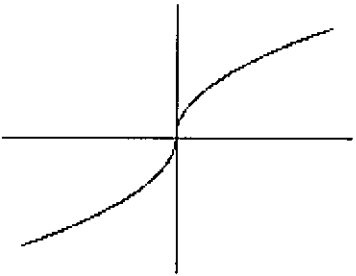


## Properties of Definite Integrals

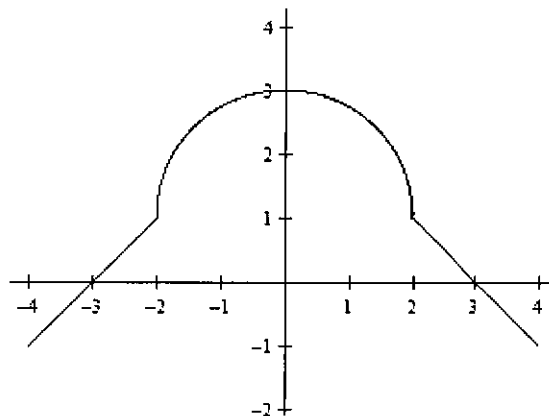
Given the integral statements, write what you think each is equivalent to. Be prepared to explain your reasoning with the rest of the class.

1. $\int_a^a f(x)dx =$
2. Given that $a < c < b$ , $\int_a^b f(x)dx =$
3. If $\int_a^b f(x)dx = K$ , then $\int_b^a f(x)dx =$
4. Given that $b < a$ , then $\int_a^b f(x)dx =$
5. If $k$ is a constant, then $\int_a^b k \cdot f(x)dx =$
6. $\int_a^b [f(x) \pm g(x)]dx =$
7. Given that $f(x)$ is an even function, $\int_{-a}^a f(x)dx =$ 
8. Given that $f(x)$ is an odd function, $\int_{-a}^a f(x)dx =$ 

If  $\int_0^3 f(x)dx = 6$  and  $\int_3^7 f(x)dx = -8$ , determine the value of each of the following integrals using the properties of definite integrals. Explain how you arrived at your answer for each.

$\int_3^0 f(x)dx$	$\int_0^7 f(x)dx$
$\int_3^3 f(x)dx$	$\int_7^3 3f(x)dx$
$\int_3^7 (2 + 3f(x))dx$	$\int_{-3}^3 f(x)dx$ , if $f(x)$ is an even function
$\int_{-3}^3 f(x)dx$ , if $f(x)$ is an odd function	

Pictured to the right is the graph of a function  $f(x)$ .



What is the value of  $\int_0^3 f(x)dx$ ?

What is the value of  $\int_0^4 f(x)dx$ ?

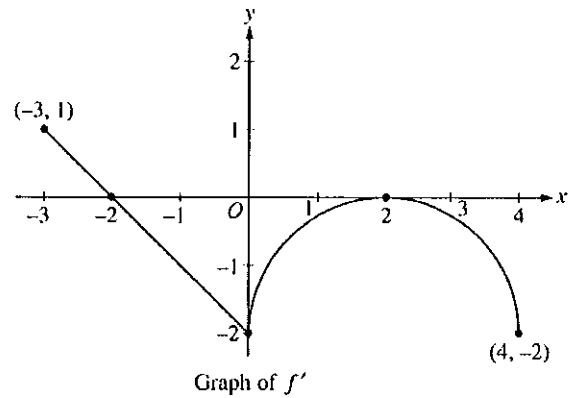
What is the value of  $\int_{-3}^3 f(x)dx$ ?

If  $F(0) = 5$ , what is the value of  $F(3)$ , where  $F$  is the anti-derivative of  $f(x)$ ?

If  $F(-2) = -2$ , what is the value of  $F(2)$ , where  $F$  is the anti-derivative of  $f(x)$ ?

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Problem #4



Let  $f$  be a function defined on the closed interval  $-3 \leq x \leq 4$  with  $f(0) = 3$ . The graph of  $f'$ , the derivative of  $f$ , consists of one line segment and a semicircle, as shown above.

- On what intervals, if any, is  $f$  increasing. Justify your reasoning.
- Find the  $x$  – coordinate of each point of inflection of the graph of  $f$  on the open interval  $-3 < x < 4$ . Justify your answer.
- Find an equation for the line tangent to the graph of  $f$  at the point  $(0, 3)$ .
- Find  $f(-3)$  and  $f(4)$ . Show the work that leads to your answers.

## Homework 6.4

Given  $\int_2^6 f(x)dx = 10$  and  $\int_2^6 g(x)dx = -2$ , find the values of each of the following definite integrals, if possible, by rewriting the given integral using the properties of integrals.

1. $\int_2^6 [f(x) + g(x)]dx$	2. $\int_2^6 [2f(x) - 3g(x)]dx$	3. $\int_2^6 2x + 2g(x)dx$
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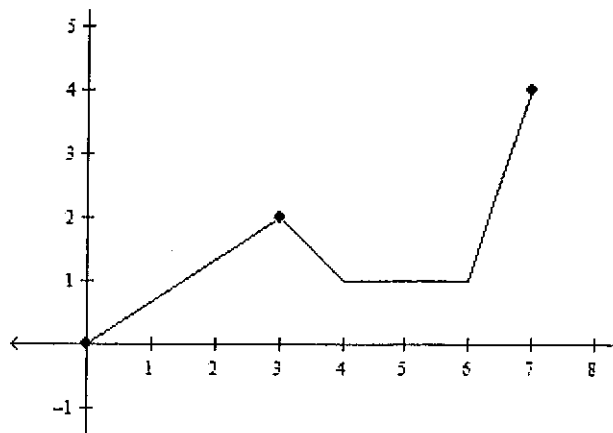
Given  $\int_{-2}^4 f(x)dx = -6$  and  $\int_{-2}^4 g(x)dx = 4$ , find the values of each of the following definite integrals. Rewrite the given integral using the properties of integrals. Then, find the value.

4. $\int_{-2}^4 [f(x) + 4]dx$	5. $\int_{-2}^4 [3g(x) + x]dx$
6. $\int_{-2}^4 \left[ \frac{1}{2}f(x) + 3x^2 \right]dx$	

Pictured below is the graph of  $f'(x)$ , the first derivative of a function  $f(x)$ . Use the graph to answer the following questions 8 –10.

Graph of  $f'(x)$

7. What is the value of  $\int_0^7 f'(x) dx$



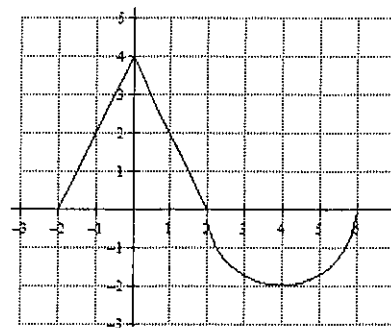
8. If  $f(0) = -3$ , what is the value of  $f(3)$ ?

9. If  $f(3) = -1$ , what is the value of  $f(7)$ ?

The graph of  $f'(x)$ , the derivative of a function,  $f(x)$ , is pictured below on the interval  $[-2, 6]$ . Write and find the value of a definite integral to find each of the indicated values of  $f(x)$  below. Also,  $f(-2) = 5$ .

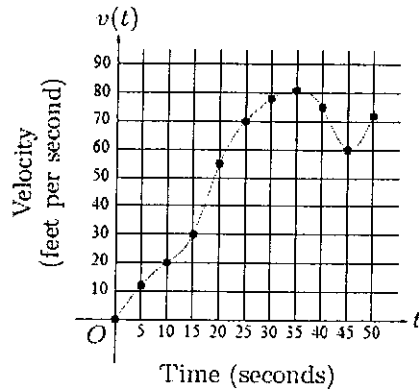
10. Find the value of  $f(0)$ .

11. Find the value of  $f(6)$ .



Graph of  $f'$

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$t$ (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity  $v(t)$ , in ft/sec, of a car traveling on a straight road, for  $0 \leq t \leq 50$ , is shown above. A table of values for  $v(t)$ , at 5 second intervals of time  $t$ , is shown to the right of the graph.
- During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
  - Find the average acceleration of the car, in  $\text{ft}/\text{sec}^2$ , over the interval  $0 \leq t \leq 50$ .
  - Find one approximation for the acceleration of the car, in  $\text{ft}/\text{sec}^2$ , at  $t = 40$ . Show the computations you used to arrive at your answer.
  - Approximate  $\int_0^{50} v(t) dt$  with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.
-

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3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function  $R$  of time  $t$ . The table above shows the rate as measured every 3 hours for a 24-hour period.

- (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate  $\int_0^{24} R(t) dt$ . Using correct units, explain the meaning of your answer in terms of water flow.
- (b) Is there some time  $t$ ,  $0 < t < 24$ , such that  $R'(t) = 0$ ? Justify your answer.

$t$ (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6