Name KEY Date ______ Class ______

Homework 3.2

In exercises 1 – 6, find the derivative of each of the following functions.

1. $f(x)=\left(\frac{x+5}{x^2+2}\right)^3$

$$f'(x)=3\left(\frac{x+5}{x^2+2}\right)^2\cdot\frac{(x^2+2)(1)-(x+5)(2x)}{(x^2+2)^2}$$

$$3\left(\frac{x+5}{x^2+2}\right)^2\left(\frac{x^2+2-2x^2-10x}{(x^2+2)^2}\right)$$

$$3\left(\frac{x+5}{x^2+2}\right)^2\left(\frac{-x^2-10x+2}{(x^2+2)^2}\right)$$

$$f'(x)=\frac{3(x+5)^2(-x^2-10x+2)}{(x^2+2)^4}$$

2. $f(x)=\sqrt{\frac{2x+3}{x-2}}$

$$f'(x)=\frac{1}{2}\left(\frac{2x+3}{x-2}\right)^{-\frac{1}{2}}\cdot\frac{(x-2)(2)-(2x+3)(1)}{(x-2)^2}$$

$$f'(x)=\frac{1}{2}\left(\frac{x-2}{2x+3}\right)^{\frac{1}{2}}\cdot\frac{\cancel{2x}-4-\cancel{2x}-3}{(x-2)^2}$$

$$\frac{-7(x-2)^{\frac{1}{2}}}{2(2x+3)^{\frac{1}{2}}(x-2)^2}$$

$$\boxed{f'(x)=\frac{-7}{2\sqrt{(2x+3)(x-2)^3}}}$$

3. $h(x)=\sqrt{x^2-3x+1}$

$$h'(x)=\frac{1}{2}(x^2-3x+1)^{-\frac{1}{2}}(2x-3)$$

$$\boxed{h'(x)=\frac{2x-3}{2\sqrt{x^2-3x+1}}}$$

4. $g(x)=\sqrt[3]{9x^2+4}$

$$g'(x)=\frac{1}{3}(9x^2+4)^{-\frac{2}{3}}\cdot 18x$$

$$\boxed{g'(x)=\frac{6x}{\sqrt[3]{(9x^2+4)^2}}}$$

5. $f(x) = x\sqrt{1-x^2}$

$$f'(x) = x \cdot \cancel{\tfrac{1}{2}}(1-x^2)^{-\frac{1}{2}}(-\cancel{2}x) + 1\,(1-x^2)^{\frac{1}{2}}$$

$$f'(x) = \frac{-x^2}{(1-x^2)^{\frac{1}{2}}} + (1-x^2)^{\frac{1}{2}}$$

$$f'(x) = \frac{-x^2 + 1 - x^2}{(1-x^2)^{\frac{1}{2}}}$$

$$\boxed{f'(x) = \frac{-2x^2+1}{\sqrt{1-x^2}}}$$

6. $p(x) = \dfrac{x}{\sqrt{x^2+1}}$

$$p'(x) = \frac{(x^2+1)^{\frac{1}{2}} \cdot 1 - x \cdot \cancel{\tfrac{1}{2}}(x^2+1)^{-\frac{1}{2}} \cdot \cancel{2}x}{x^2+1}$$

$$p'(x) = \frac{(x^2+1)^{\frac{1}{2}} - \dfrac{x^2}{(x^2+1)^{\frac{1}{2}}}}{x^2+1}$$

$$p'(x) = \frac{\dfrac{x^2+1-x^2}{(x^2+1)^{\frac{1}{2}}}}{x^2+1}$$

$$p'(x) = \frac{1}{(x^2+1)^{\frac{1}{2}}(x^2+1)}$$

$$p'(x) = \frac{1}{\sqrt{(x^2+1)^3}}$$

For exercises 7 and 8, find the value of the derivative of the function at the given point.

7. $g(\theta) = \frac{1}{4}\sin^2 2\theta$ when $\theta = \pi$

$$g'(\theta) = \frac{1}{\cancel{2}} \sin 2\theta \cdot \cos 2\theta \cdot \cancel{2}$$

$$g'(\theta) = \sin 2\theta \cdot \cos 2\theta$$

$$g'(\pi) = \sin 2\pi \cdot \cos 2\pi$$

$$1 \cdot 0$$

$$g'(\pi) = 0$$

8. $f(\theta) = \sin 2\theta \cos 2\theta$ when $\theta = \frac{\pi}{4}$

$$f'(\theta) = \sin 2\theta(-2\sin 2\theta) + 2\cos 2\theta \cos 2\theta$$

$$f'(\theta) = -2\sin^2(2\theta) + 2\cos^2(2\theta)$$

$$f'\left(\frac{\pi}{4}\right) = -2\sin^2\left(2 \cdot \frac{\pi}{4}\right) + 2\cos^2\left(2 \cdot \frac{\pi}{4}\right)$$

$$f'\left(\frac{\pi}{4}\right) = -2(1)^2 + 0$$

$$f'\left(\frac{\pi}{4}\right) = -2$$

Use the table below to complete exercises 16 – 17.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–2	1	–1	2	4
–1	3	–2	1	1
0	–1	2	–2	–3

16. If $H(x)=\sqrt{f(x)\cdot g(x)}$, is the graph of $H(x)$ increasing or decreasing when $x=-1$? Give a reason for your answer.

$$H'(x) = \frac{1}{2}\left[f(x)\cdot g(x)\right]^{-\frac{1}{2}}\cdot\left[f(x)\cdot g'(x) + f'(x)\cdot g(x)\right]$$

$$H'(-1) = \frac{1}{2}\left[f(-1)\cdot g(-1)\right]^{-\frac{1}{2}}\cdot\left[f(-1)\cdot g'(-1) + f'(-1)\cdot g(-1)\right]$$

$$H'(-1) = \frac{1}{2}\left[3\cdot 1\right]^{-\frac{1}{2}}\cdot\left[3\cdot 1 + (-2)(1)\right] = \frac{1}{2\sqrt{3}}$$

$H(x)$ IS INCREASING B/C $H'(-1) > 0$

17. If $P(x) = (2f(x)+g(x))^{\frac{2}{3}}$, what is the value of $P'(0)$?

$$P'(x) = \frac{2}{3}\left(2f(x)+g(x)\right)^{-\frac{1}{3}}\cdot\left(2f'(x)+g'(x)\right)$$

$$P'(0) = \frac{2\left(2f'(0)+g'(0)\right)}{3\left(2f(0)+g(0)\right)^{\frac{1}{3}}} = \frac{2\left[2(2)-3\right]}{3\left[2(-1)-2\right]^{\frac{1}{3}}} = \frac{2}{3\sqrt[3]{\cancel{-4}}} = -\frac{2}{3\sqrt[3]{4}}$$

18. Find the equation of the normal line to the graph of $h(x) = \tan(3x)$ when $x = \frac{\pi}{12}$.

$$h'(x) = \sec^2(3x)\cdot 3$$

$$h'\left(\tfrac{\pi}{12}\right) = 3\sec^2\left(3\cdot\tfrac{\pi}{12}\right)$$

$$h'\left(\tfrac{\pi}{12}\right) = 3\sec^2\left(\tfrac{\pi}{4}\right)$$

$$h'\left(\tfrac{\pi}{12}\right) = 3\left(\tfrac{2}{\sqrt{2}}\right)^2$$

$$h'\left(\tfrac{\pi}{12}\right) = 3\cdot\frac{4}{2} = 6 \leftarrow \text{S.O.T.}$$

$$-\frac{1}{6} \leftarrow \text{S.O.N}$$

$$h\left(\tfrac{\pi}{12}\right) = \tan\left(3\cdot\tfrac{\pi}{12}\right)$$

$$h\left(\tfrac{\pi}{12}\right) = 1$$

$$y - 1 = -\frac{1}{6}\left(x - \frac{\pi}{12}\right)$$

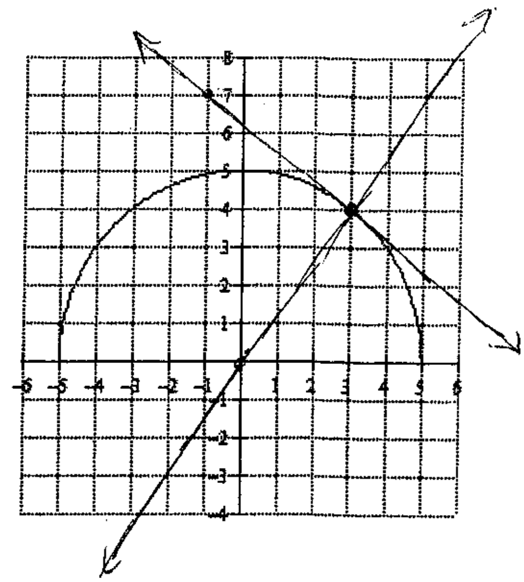
The graph of the function $f(x) = \sqrt{25 - x^2}$ is pictured to the right. Use this function to complete exercises 9 – 11.

9. Find the values of $f(3)$ and $f'(3)$.

$f(3) = \sqrt{25-9} = 4$

$f'(3) = \frac{1}{2}(25-3^2)^{-\frac{1}{2}} \cdot (-2(3))$

$f'(3) = \frac{1}{4} \cdot 3 = -\frac{3}{4}$



10. Find the equation of the line tangent to the graph of $f(x)$ when $x = 3$ and graph this line on the grid with $f(x)$.

$y - 4 = -\frac{3}{4}(x-3)$

11. Find the equation of the normal line to the graph of $f(x)$ when $x = 3$ and graph this line on the grid with $f(x)$.

$y - 4 = \frac{4}{3}(x-3)$

12. Find the following limit. Explain the reasoning that you used to arrive at your answer.

$$\lim_{h \to 0} \frac{\cos 3(x+h) - \cos 3x}{h}$$

$f(x) = \cos 3x$

$f'(x) = -3 \sin 3x$

13. If $f(x) = \tan 4x$, then $f'(x) =$ $4 \sec^2(4x)$.

14. If $f(\theta) = \sec 5\theta$, then $f'(x) =$ $5 \tan(5\theta) \sec(5\theta)$

15. If $f(\theta) = \csc 2\theta$, then $f'(x) =$ $-2 \cot(2\theta) \csc(2\theta)$