1) $$\lim_{x\to 2} \frac{3x^2-4x-4}{x^2-4} = \lim_{x\to 2} \frac{6x-4}{2x} = \frac{8}{4} = \boxed{2}$$

2) $$\lim_{x\to \infty} \frac{x^3-2x^2}{4x^3-x-3} = \lim_{x\to\infty} \frac{3x^2-4x}{12x^2-1} = \lim_{x\to\infty} \frac{6x-4}{24x} = \lim_{x\to\infty} \frac{6}{24} = \boxed{\frac{1}{4}}$$

3) $$\lim_{\theta\to \frac{\pi}{2}} \frac{1-\sin\theta}{1+\cos(2\theta)} = \lim_{\theta\to\frac{\pi}{2}} \frac{-\cos\theta}{-2\sin(2\theta)} = \lim_{\theta\to\frac{\pi}{2}} \frac{\sin\theta}{-4\cos(2\theta)} = \boxed{\frac{1}{4}}$$

4) $$\lim_{x\to 1} \frac{x-1}{\ln x - \sin(\pi x)} = \lim_{x\to 1} \frac{1}{\frac{1}{x} - \pi\cos(\pi x)} = \boxed{\frac{1}{1+\pi}}$$

5) $$\lim_{x\to 0} \frac{5x\sin x}{x^2-\sin(2x)} = \lim_{x\to 0} \frac{5x\cos x + 5\sin x}{2x - 2\cos(2x)} = \frac{0}{-2} = \boxed{0}$$

6) $$\lim_{x\to\infty} \frac{2x^2+3x}{x^3+x+1} = \lim_{x\to\infty} \frac{4x+3}{3x^2+1} = \lim_{x\to\infty} \frac{4}{6x} = \boxed{0}$$

7) $$\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} = \lim_{x\to 2} \frac{\frac{1}{2}(x+2)^{-\frac{1}{2}}}{1} = \frac{1}{2}\left(\frac{1}{2}\right) = \boxed{\frac{1}{4}}$$

8) $$\lim_{x\to 0} \frac{\sqrt{x+1}-1-\frac{x}{2}}{x^2} = \lim_{x\to 0} \frac{\frac{1}{2}(x+1)^{-\frac{1}{2}} - \frac{1}{2}}{2x} = \frac{-\frac{1}{4}(x+1)^{-\frac{3}{2}}}{2} = \boxed{-\frac{1}{8}}$$

9) $$\lim_{x\to 1} \frac{x\ln x}{x^2-1} = \lim_{x\to 1} \frac{x\left(\frac{1}{x}\right) + \ln x}{2x} = \boxed{\frac{1}{2}}$$

10) $$\lim_{x\to 0} \frac{e^x-1}{2x} = \frac{e^x}{2} = \boxed{\frac{1}{2}}$$

11) $$\lim_{x\to 3} \frac{9-x^2}{\cos\left(\frac{\pi}{2}x\right)} = \lim_{x\to 3} \frac{-2x}{-\frac{\pi}{2}\sin\left(\frac{\pi}{2}x\right)} = \frac{-6}{+\frac{\pi}{2}} = \boxed{\frac{-12}{\pi}}$$

PARTICLE MOTION

$f(t) = t^3 - 6t^2 + 9t$ ← POSITION

a) $v(t) = 3t^2 - 12t + 9$

$v(4) = 3(4)^2 - 12(4) + 9$

$\boxed{v(4) = 9 \text{ METERS/SEC}}$

b) $v(t) = 3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-3)(t-1) = 0$

$\boxed{t = 3, 1}$

c) $v(t)$

0		1		3		4
	0.5		2		3.5	
	+		−		+	

MOVING FORWARD ON $(0,1) \cup (3,4)$
MOVING BACKWARD ON $(1,3)$

d) $a(t) = 6t - 12 = 0$

$6t = 12$

$t = 2$

$a(t)$

0		2		4
	1		3	
	−		+	

SPEEDING UP ON $(1,2) \cup (3,4)$
SLOWING DOWN ON $(0,1) \cup (2,3)$

e) $|p(0) - p(1)| + |p(1) - p(3)| + |p(3) - p(4)|$

$|0 - 4| + |4 - 0| + |0 - 4|$

$\boxed{12 \text{ METERS}}$

f) $a(3) = 6(3) - 12$

$\boxed{6 \text{ METERS}/\text{SEC}^2}$

g) $f(0) = 0$

$f(1) = 4$

$f(3) = 0$

$f(4) = 4$

$\boxed{\text{MAX HEIGHT IS } 4 \text{ METERS}}$

h) $f(4) = 4 \qquad f(0) = 0$

$4 - 0$

$\boxed{4 \text{ METERS}}$

i) $v(t) = 3t^2 - 12t + 9 = 9$

$3t^2 - 12t = 0$

$3t(t-4) = 0$

$\boxed{t = 0, 4 \text{ SECONDS}}$

j) AVG. VELOCITY $= \dfrac{f(0) - f(4)}{0 - 4} = \dfrac{0-4}{-4} = \boxed{1 \text{ METER}/\text{SEC}}$

1) $s(t) = t^3 - 3t + 3 \qquad [0, 6]$

a) $v(t) = 3t^2 - 3$

$v(3) = 3(3)^2 - 3 = 24$ METERS/SEC

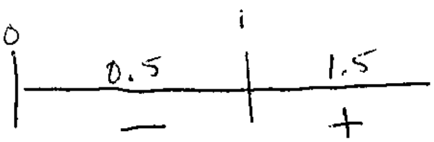
b) $v(t) = 3t^2 - 3 = 0$

$3(t^2 - 1) = 0$

$3(t+1)(t-1) = 0$

$t = \pm 1$

$\boxed{t = 1 \text{ SECOND}}$

c) $v(t)$ 

MOVING LEFT ON $(0, 1)$

MOVING RIGHT ON $(1, 6)$

d) $a(t) = 6t$

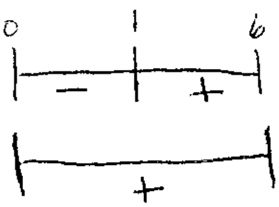
$a(1) = 6(1)$

$\boxed{6 \text{ METERS/SEC}^2}$

e) NET DISTANCE $= |s(0) - s(6)| = |3 - 201| = \boxed{198 \text{ METERS}}$

TOTAL DISTANCE $= |s(0) - s(1)| + |s(1) - s(6)|$

$|3 - 1| + |1 - 201| = \boxed{202 \text{ METERS}}$

f) $v(t)$, $a(t)$ 

SPEEDING UP ON $(1, 6)$

SLOWING DOWN ON $(0, 1)$

g) $a(t) = 0$ @ $t = 0$, $\boxed{v(0) = -3 \text{ METERS/SEC}}$

6) ENRICHMENT

$s(t) = S_0 + V_0 t - \frac{1}{2} g t^2$, $v(t) = \frac{ds}{dt} = V_0 - gt$

a) $s(t) = 6 + 300t - \frac{1}{2}(32)t^2$

$s(t) = -16t^2 + 300t + 6$

b) $v(t) = -32t + 300$

c) $v(2) = -32(2) + 300$

$v(2) = 236$ ft/SEC

d) $v(t) = -32t + 300 = 0$

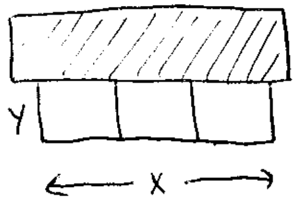
$-32t = -300$

$t = \frac{75}{8}$ SECONDS

$s\left(\frac{75}{4}\right) = -16\left(\frac{75}{8}\right)^2 + 300\left(\frac{75}{8}\right) + 6$

$-\frac{75^2}{4} + \frac{75^2}{2} + 6 = \frac{-75^2 + 2(75)^2 + 24}{4} \approx 1412$ FT

1)



- AREA IS BEING MAXIMIZED
- $A = xy$
- $200 = x + 4y$

$200 - 4y = x$

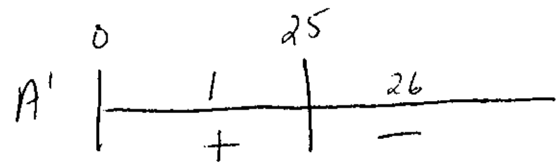
$A = (200 - 4y)\,y$

$A = 200y - 4y^2$

$A' = 200 - 8y = 0$

$-8y = -200$

$y = 25$



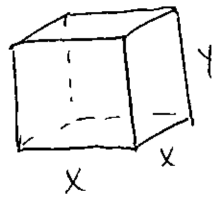
AREA IS MAXIMIZED WHEN $y = 25$

$200 = x + 4(25)$

$100 = x$

MAX AREA $= 100(25) = \boxed{2500 \text{ FT}^2}$

2) SURFACE AREA IS BEING MINIMIZED



$$A = x^2 + 4xy$$

$$500 = x^2 y$$

$$\frac{500}{x^2} = y$$

$$A = x^2 + 4x\left(\frac{500}{x^2}\right)$$

$$A = x^2 + 2000x^{-1}$$

$$A' = 2x - 2000x^{-2} = 0$$

$$2x - \frac{2000}{x^2} = 0$$

$$\frac{2x^3 - 2000}{x^2} = 0$$

$$2x^3 - 2000 = 0$$

$$2x^3 = 2000$$

$$x^3 = 1000$$

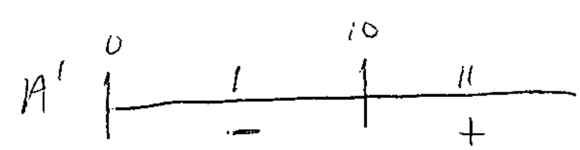
$$x = 10$$

$x^2 = 0$

$x = 0$ DOESN'T MAKE SENSE

$$500 = 10^2 y$$

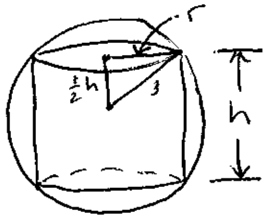
$$5 = y$$



AREA IS MINIMIZED WHEN $x = 10$

DIMENSIONS ARE $10 \times 10 \times 5$

3) VOLUME IS BEING MAXIMIZED



$$V = \pi r^2 h$$

$$9 = r^2 + \left(\tfrac{1}{2}h\right)^2$$

$$9 = r^2 + \tfrac{1}{4}h^2$$

$$9 - \tfrac{1}{4}h^2 = r^2$$

$$V = \pi\left(9 - \tfrac{1}{4}h^2\right)h$$

$$V = \left(9\pi - \tfrac{\pi}{4}h^2\right)h$$

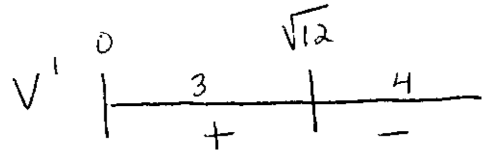
$$V = 9\pi h - \tfrac{\pi}{4}h^3$$

$$V' = 9\pi - \tfrac{3\pi}{4}h^2 = 0$$

$$9\pi = \tfrac{3\pi}{4}h^2$$

$$12 = h^2$$

$$\sqrt{12} = h$$



VOLUME IS MAXIMIZED WHEN $h = \sqrt{12}$

$$9 = r^2 + \tfrac{1}{4}\left(\sqrt{12}\right)^2$$

$$9 = r^2 + 3$$

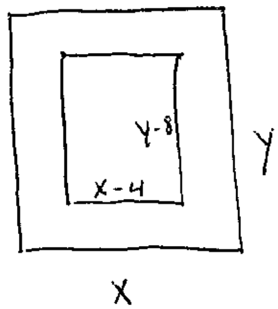
$$6 = r^2$$

$$\sqrt{6} = r$$

MAX VOLUME $= \pi\left(\sqrt{6}\right)^2\left(\sqrt{12}\right)$

$$\boxed{\begin{array}{c} = 6\pi\sqrt{12}\ \text{in}^3 \\ \text{OR} \\ 12\pi\sqrt{3}\ \text{in}^3 \end{array}}$$

4) AREA IS BEING MINIMIZED



$A = xy$

$50 = (x-4)(y-8)$

$\frac{50}{x-4} + 8 = y$

$A = x\left(\frac{50}{x-4} + 8\right)$

$A = \frac{50x}{x-4} + 8x$

$A' = \frac{50(x-4) - 50x}{(x-4)^2} + 8$

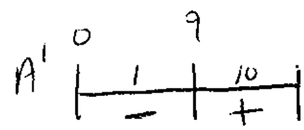
$A' = \frac{-200}{(x-4)^2} + 8 = 0$

$\frac{-200}{(x-4)^2} = -8$

$25 = (x-4)^2$

$5 = x-4$

$9 = x$



AREA IS BEING MINIMIZED WHEN $x=9$

$\frac{50}{9-4} + 8 = y$

$18 = y$

MIN AREA OF POSTER $= 9(18)$

$\boxed{9_{\text{IN}} \times 18_{\text{IN}}}$

DERIVATIVES OF INVERSE TRIG FUNCTIONS

a) $\frac{d}{dx}\left(x^3 \arcsin(3x)\right) = x^3 \cdot \frac{1}{\sqrt{1-(3x)^2}} \cdot 3 + 3x^2 \arcsin(3x)$

$$\boxed{\frac{3x^3}{\sqrt{1-9x^2}} + 3x^2 \arcsin(3x)}$$

b) $\frac{d}{dx}\left(\frac{\sqrt{x}}{\arcsin(x)}\right) = \frac{\arcsin(x) \cdot \frac{1}{2}x^{-\frac{1}{2}} - \sqrt{x} \cdot \frac{1}{\sqrt{1-x^2}}}{\arcsin^2(x)}$

$$\frac{\frac{\arcsin(x)}{2\sqrt{x}} - \frac{\sqrt{x}}{\sqrt{1-x^2}}}{\arcsin^2(x)} = \frac{\frac{\arcsin(x)\sqrt{1-x^2} - 2\sqrt{x-x^3}}{2\sqrt{x-x^3}}}{\arcsin^2(x)}$$

$$\boxed{\frac{\arcsin(x)\sqrt{1-x^2} - 2\sqrt{x-x^3}}{2\arcsin^2(x)\sqrt{x-x^3}}}$$

c) $\frac{d}{dx}\left[\ln(\arcsin(e^x))\right] = \frac{1}{\arcsin(e^x)} \cdot \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x = \frac{e^x}{\arcsin(e^x)\sqrt{1-e^{2x}}}$

d) $\frac{d}{dx}\left[\arcsin(\cos x)\right] = \frac{1}{\sqrt{1-\cos^2 x}} \cdot (-\sin x) = \frac{-\sin x}{\sqrt{1-\cos^2 x}}$

a) $\frac{d}{dx}\left[\arctan(e^x)\right] = \frac{1}{1+(e^x)^2} \cdot e^x = \boxed{\frac{e^x}{1+e^{2x}}}$

b) $\frac{d}{dx}\left[e^x \arctan(x)\right] = e^x \cdot \frac{1}{1+x^2} + e^x \arctan(x)$

$$\boxed{\frac{e^x}{1+x^2} + e^x \arctan(x)}$$

c) $\frac{d}{dx}\left[\sin(\arctan(x))\right] = \cos(\arctan(x)) \cdot \frac{1}{1+x^2}$

$$\frac{1}{\sqrt{1+x^2}} \cdot \frac{1}{1+x^2} = \boxed{\frac{1}{(1+x^2)^{\frac{3}{2}}}}$$

d) $\frac{d}{dx}\left[\arctan(\arcsin(x^2))\right] = \frac{1}{1+(\arcsin(x^2))^2} \cdot \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$

$$\frac{2x}{\left[1+\arcsin^2(x^2)\right]\sqrt{1-x^4}}$$