The Fundamental Theorem of Calculus, Particle Motion, and Average Value

Three Things to Always Keep In Mind:

(1) $\int_a^b v(t)dt = p(b) - p(a)$, where $v(t)$ represents the velocity and $p(t)$ represents the position.

(2) $\int_a^b v(t)dt$ = The Net Distance the particle travels on the interval from $t = a$ to $t =$ b. If $v(t) > 0$ on the interval (a, b), then it also represents the Total Distance.

(3) $\int_a^b |v(t)|dt$ = The Total Distance the particle travels on the interval (a, b), whether or not $v(t) > 0$. To be safe, always do this integral when asked to find total distance when given velocity.

1. The velocity of a particle that is moving along the x – axis is given by the function $v(t) = 3t^2 + 6$. (This is a non-calculator active question.)

   a. If the position of the particle at $t = 4$ is 72, what is the position when $t = 2$?

   b. What is the total distance the particle travels on the interval $t = 0$ to $t = 7$?

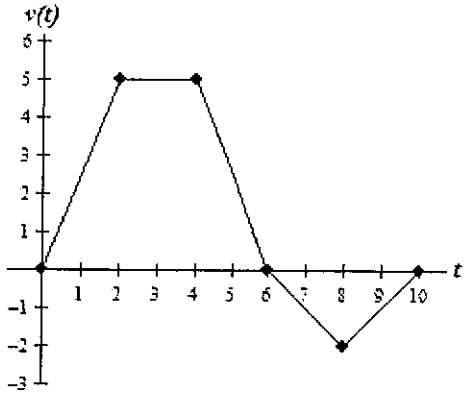
2. The velocity of a particle that is moving along the $x-$ axis is given by the function $v(t) = 0.5e^t(t-2)^3$. (This is a calculator active question.)

   a. If the position of the particle at $t = 1.5$ is 2.551, what is the position when $t = 3.5$?

   b. What is the total distance that the object travels on the interval $t = 1$ to $t = 5$?

The graph of the velocity, measured in feet per second, of a particle moving along the $x-$ axis is pictured below. The position, $p(t)$, of the particle at $t = 8$ is 12. Use the graph of $v(t)$ to answer the questions that follow.

a. What is the position of the particle at $t = 3$?



b. What is the acceleration when $t = 5$?

c. What is the net distance the particle travels from $t = 0$ to $t = 10$?

d. What is the total distance the particle travels from $t = 0$ to $t = 10$?

t	0	3	6	9	12	15	18
$V(t)$	2.3	2.7	2.0	1.3	1.0	1.7	2.1

The table above shows values of the velocity, $V(t)$ in meters per second, of a particle moving along the x – axis at selected values of time, t seconds.

a. What does the value of $\int_0^{18} V(t)dt$ represent?

b. Using a left Riemann sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

c. Using a right Riemann sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

d. Using a midpoint Riemann sum of 3 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

e. Using a trapezoidal sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

f. Find the average acceleration of the particle from $t = 3$ to $t = 9$. For what value of t, in the table, is this average acceleration approximately equal to $v'(t)$? Explain your reasoning.

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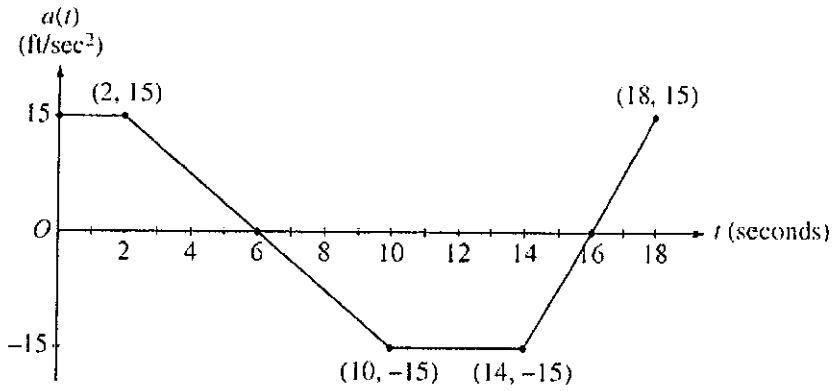
Problem #3



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

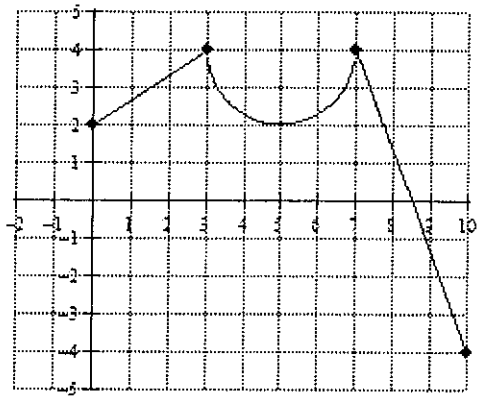
(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

Name_____ Date_____ Class_____

Homework 6.5

The graph to the right represents the velocity, $v(t)$ in meters per second, of a particle that is moving along the $x-$axis on the time interval $0 \le t \le 10$. The initial position of the particle at time $t = 0$ is 12.



1. On what interval(s) of time is the particle moving to the left and to the right? Justify your answer.

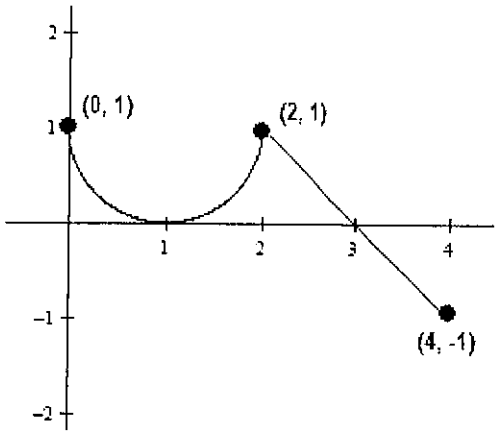
2. What is the total distance that the particle has traveled on the time interval $0 \le t \le 7$. Leave your answer in terms of π. Indicate units of measure.

3. What is the net distance that the particle travels on the interval $5 \le t \le 10$? Round your answer to the nearest thousandth. Indicate units of measure.

4. What is the acceleration of the particle at time $t = 2$? Indicate units of measure.

5. What is the position of the particle at time $t = 5$? Indicate units of measure.

Pictured to the right is the graph of a function which represents a particle's velocity on the interval [0, 4]. Answer the following questions.



6. For what values is the particle moving to the right? Justify your answer.

7. For what values is the particle moving to the left? Justify your answer.

8. For what values is the speed of the particle increasing? Justify your answer.

9. For what values is the speed of the particle decreasing? Justify your answer.

10. What is the net distance that the particle travels on the interval [0, 4]?

11. What is the total distance that the particle travels on the interval [0, 4]?

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity, v, measured in feet per second, and acceleration, a, measured in feet per second per second, are continuous and differentiable functions on $0 \le t \le 60$. The table below shows selected values of these functions.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

12. Using appropriate units, explain the meaning of $\int_0^{60} |v(t)|dt$ in terms of the car's motion. Approximate this integral using a midpoint approximation with three subintervals as determined by the table.

13. Using appropriate units, explain the meaning of $\int_{15}^{50} a(t)dt$ in terms of the car's motion. Find the exact value of the integral.

14. Is there a value of t such that $a'(t) = 0$? If so, identify an interval on which such a value of t exists? Justify your reasoning.

15. Using appropriate units, approximate the value of $v'(31)$. What does this value say about the motion of the car at $t = 31$.

16. Using appropriate units, find the value and explain the meaning of $\frac{1}{35}\int_{25}^{60} a(t)dt$.

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Problem #3

A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

(a) On the axes provided, sketch the graph of $v(t)$ for $0 \le t \le 16$.

(b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

(d) Is there any time t, $0 < t \le 16$, at which the particle returns to the origin? Justify your answer.