I. Limits and Continuity

A. $f(x) = 2x^2 + 1$

$$\lim_{x\to 0} \frac{f(x) - f(0)}{x^2} = \lim_{x\to 0} \frac{2x^2 + 1 - (2(0)^2 + 1)}{x^2} = \lim_{x\to 0} \frac{2\cancel{x^2}}{\cancel{x^2}} = \boxed{2}$$

B. $$\lim_{x\to\infty} \frac{3x^2 + 2x^5 - x}{x^2 - 3x^5} = \boxed{\frac{3}{4}}$$

C. $$g(x) = \begin{cases} k \sin x + x^2, & 0 \le x < \frac{\pi}{4} \\ x + 2, & \frac{\pi}{4} \le x \le e \\ m \ln e^x, & e < x \le 2\pi \end{cases}$$

$$\lim_{x\to \frac{\pi}{4}^-} g(x) = \lim_{x\to\frac{\pi}{4}^+} g(x) \qquad \lim_{x\to e^-} g(x) = \lim_{x\to e^+} g(x)$$

$$k \sin\left(\tfrac{\pi}{4}\right) + \left(\tfrac{\pi}{4}\right)^2 = \tfrac{\pi}{4} + 2 \qquad e + 2 = m \ln e^e$$

$$\frac{\sqrt{2}}{2} k + \frac{\pi^2}{16} = \frac{\pi}{4} + 2 \qquad e + 2 = m \cdot e$$

$$\frac{\sqrt{2}}{2} k = \frac{\pi}{4} + 2 - \frac{\pi^2}{16} \qquad \frac{e+2}{e} = m$$

$$k = \frac{\frac{\pi}{4} + 2 - \frac{\pi^2}{16}}{\frac{\sqrt{2}}{2}} \qquad \boxed{m = 1.736}$$

$$\boxed{k = 3.067}$$

II. UNDERSTANDING THE DERIVATIVE

A. $f(x) = 4x^3 - 5x + 3$

<u>POT</u>

$f(-1) = 4(-1)^3 - 5(-1) + 3$

$f(-1) = -4 + 5 + 3$

$f(-1) = 4$

$(-1, 4)$

<u>SOT</u>

$f'(x) = 12x^2 - 5$

$f'(-1) = 12(-1)^2 - 5$

$f'(-1) = 7$

$m = 7$

$$\boxed{y - 4 = 7(x+1)}$$

OR

$$\boxed{y = 7x + 11}$$

B. $h(x) = \dfrac{3x}{\sqrt[3]{x}} = 3x \cdot x^{-\frac{1}{3}} = 3x^{\frac{2}{3}}$

$h'(x) = 2x^{-\frac{1}{3}} = \dfrac{2}{\sqrt[3]{x}}$

$h'(8) = \dfrac{2}{\sqrt[3]{8}} = \dfrac{2}{2} = 1 \leftarrow$ SOT

$$\boxed{\text{SON} = -1}$$

C. $3y + 2 = 2(3 - x)$

$3y = 4 - 2x$

$y = -\frac{2}{3}x + \frac{4}{3}$

SON $= -\frac{2}{3}$

SOT $= \frac{3}{2}$

$$\boxed{\therefore f'(a) = \frac{3}{2}}$$

III. RULES OF THE DERIVATIVE

A. $f(x) = \begin{cases} 2x - x^2 & x \leq 1 \\ x^2 + kx + p & x > 1 \end{cases}$ $\qquad f'(x) = \begin{cases} 2 - 2x & x < 1 \\ 2x + k & x > 1 \end{cases}$

<u>CONTINUOUS</u>

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$2(1) - 1^2 = 1^2 + k(1) + p$$

$$1 = 1 + k + p$$

$$0 = k + p$$

<u>DIFFERENTIABLE</u>

$$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$$

$$2 - 2(1) = 2(1) + k$$

$$0 = 2 + k$$

$$-2 = k$$

$$\boxed{\begin{matrix} k = -2 \\ p = 2 \end{matrix}}$$

B. $y = \left(\dfrac{x^3 - 2}{2x^5 - 1}\right)^4$

$$\frac{dy}{dx} = 4\left(\frac{x^3 - 2}{2x^5 - 1}\right)^3 \cdot \left(\frac{(2x^5 - 1)(3x^2) - (x^3 - 2)(10x^4)}{(2x^5 - 1)^2}\right)$$

$$4\left(\frac{1^3 - 2}{2(1)^5 - 1}\right)^3 \left(\frac{(2(1)^5 - 1)(3(1)^2) - (1^3 - 2)(10(1)^4)}{(2(1)^5 - 1)^2}\right)$$

$$4\left(\frac{-1}{1}\right)^3 \left(\frac{(1)(3) - (-1)(10)}{1^2}\right)$$

$$-4(13)$$

$$\boxed{-52}$$

C. $f(x) = \ln\left(x + 4 + e^{-3x}\right)$

$$f'(x) = \frac{1}{x+4+e^{-3x}} \cdot \left(1 - 3e^{-3x}\right)$$

$$f'(x) = \frac{1-3e^{-3x}}{x+4+e^{-3x}}$$

$$f'(0) = \frac{1-3e^{-3(0)}}{0+4+e^{-3(0)}} = \frac{-2}{5}$$

$$\boxed{-\frac{2}{5}}$$

IV. APPLICATIONS OF THE DERIVATIVE, PART I.

A. $e^{2y} + xy = 3x^2$

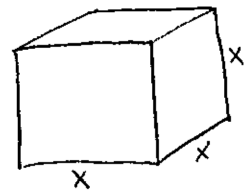
$$2e^{2y}\frac{dy}{dx} + x\frac{dy}{dx} + (1)y = 6x$$

$$\frac{dy}{dx}\left(2e^{2y} + x\right) = 6x - y$$

$$\frac{dy}{dx} = \frac{6x-y}{2e^{2y}+x}$$

$$\left.\frac{dy}{dx}\right|_{(-1,2)} = \frac{6(-1)-2}{2e^{2(2)}-1} = \boxed{\frac{-8}{2e^{4}-1}}$$

B. $\dfrac{dV}{dt} = -36 \text{ in}^3/\text{min}$



$V = x^3$

$S = 6x^2$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$-36 = 3(2)^2 \frac{dx}{dt}$$

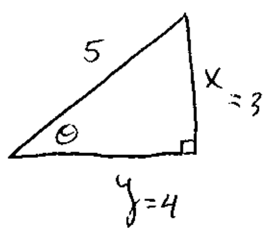
$$-3 = \frac{dx}{dt}$$

$$\frac{dS}{dt} = 12x \frac{dx}{dt}$$

$$\frac{dS}{dt} = 12(2)(-3)$$

$$\boxed{\frac{dS}{dt} = -72 \text{ in}^2/\text{min}}$$

C.



$$\cos\theta = \frac{y}{5}$$

$$-\sin\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dy}{dt}$$

$$-\left(\frac{3}{5}\right)(3) = \frac{1}{5}\frac{dy}{dt}$$

$$-\frac{9}{5} = \frac{1}{5}\frac{dy}{dt}$$

$$\boxed{-9 \text{ units/min} = \frac{dy}{dt}}$$

V. APPLICATIONS OF DERIVATIVE, PART II.

A. $x(t) = \sin t - \cos t$

$$v(t) = \cos t + \sin t = 0$$

$$\cos t = -\sin t$$

$$t = \frac{3\pi}{4}$$

$$a(t) = -\sin t + \cos t$$

$$a\left(\frac{3\pi}{4}\right) = -\sin\left(\frac{3\pi}{4}\right) + \cos\left(\frac{3\pi}{4}\right)$$

$$-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = \frac{-2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$$

B. $v(t) = 6t - t^2$

I. $v(7) = 6(7) - 7^2 = -7 < 0 \therefore$ I. IS TRUE

II. $v(t) = 6t - t^2 = 0$

$$t(6-t) = 0$$

$$t = 0, 6$$

$0 \quad + \quad 6 \quad - \quad \infty$

THE PARTICLE CHANGES DIRECTIONS AT $t = 6$, $\therefore |p(2) - p(8)|$ WOULD BE THE NET DISTANCE, THE TOTAL DISTANCE WOULD BE $|p(2) - p(6)| + |p(6) - p(8)|$

SO II IS FALSE

C. $p(t) = 3t + 4.1 \sin t$

$$v(t) = 3 + 4.1 \cos t$$

$$\int_3^5 |v(t)|\,dt = \boxed{2.887}$$

VI. BASIC INTEGRATION

A. $\int_{-1}^{0} \left[2f'(x) + 3f''(x)\right] dx$

$$2f(x) + 3f'(x) \Big]_{-1}^{0}$$

$$\left[2f(0) + 3f'(0)\right] - \left[2f(-1) + 3f'(-1)\right]$$

$$\left[2(3) + 3(3)\right] - \left[2(1) + 3(0)\right]$$

$$15 - 2$$

$$\boxed{13}$$

B. $\int_{0}^{6} f(x)\, dx \approx \frac{1}{2}(2)\left[4 + 2k + 2(8) + 12\right] = 52$

$$32 + 2k = 52$$

$$2k = 20$$

$$\boxed{k = 10}$$

C. $g(x) = x^2 - 3x + 4 \qquad f(x) = g'(x)$

$$\int_{1}^{3} f(x)\, dx = g(3) - g(1)$$

$$4 - 2$$

$$\boxed{2}$$

VII. APPLICATIONS OF INTEGRATION

A. $f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt$

$$f'(x) = \sqrt{(2x)^2 - (2x)} \cdot 2$$

$$f'(2) = \sqrt{16 - 4} \cdot 2$$

$$\boxed{f'(2) = 2\sqrt{12}}$$

B. $A = \int_0^4 f(x)\, dx + \int_4^6 g(x)\, dx$

$$A = \int_0^4 x^{\frac{1}{2}}\, dx + \int_4^6 6 - x\, dx$$

$$A = \frac{2}{3}x^{\frac{3}{2}} \Big|_0^4 + 6x - \frac{1}{2}x^2 \Big|_4^6$$

$$A = \left[\frac{2}{3}(4)^{\frac{3}{2}} - \frac{2}{3}(0)^{\frac{3}{2}}\right] + \left[\left(6(6) - \frac{1}{2}(6)^2\right) - \left(6(4) - \frac{1}{2}(4)^2\right)\right]$$

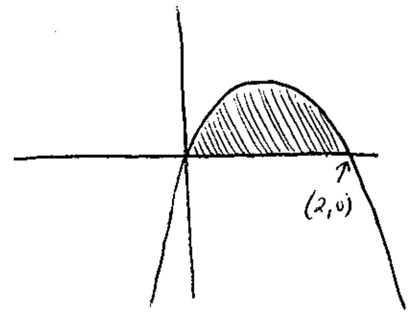
$$A = \frac{16}{3} + [18 - 16]$$

$$A = \frac{16}{3} + \frac{6}{3}$$

$$\boxed{A = \frac{22}{3}}$$

C. $V = \frac{\sqrt{3}}{4}\int_0^2 (2x - x^2 - 0)^2 \, dx$

$\boxed{0.462}$



D. $V = \pi \int_0^{3.5} (e^{x-3} - 7)^2 - (2 + \sin x - 7)^2 \, dy$

$\boxed{253.689}$