B CALCULUS
HAPTER 1 REVIEW

NAME KC

NO CALCULATORS!
SHOW YOUR WORK!

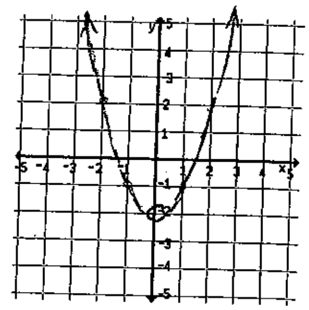
Sketch a graph, then evaluate the limits.

1. $h(x) = \dfrac{x^3 - 2x}{x} = \dfrac{\cancel{x}(x^2-2)}{\cancel{x}}$

   a. $\lim\limits_{x \to 3} h(x) =$ 7

   b. $\lim\limits_{x \to 0} h(x) = -2$

1. Graph:

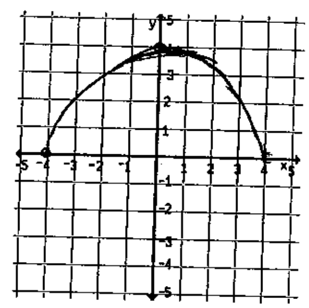


2. $y = \sqrt{16 - x^2}$

   a. $\lim\limits_{x \to -4^+} \sqrt{16 - x^2} = 0$

   b. $\lim\limits_{x \to -4^-} \sqrt{16 - x^2} =$ DNE

2. Graph:



Evaluate the limits analytically.

3. $\lim\limits_{x \to 2} (4x^2 + 5x - 6)$

   $4(2)^2 + 5(2) - 6$

   $16 + 10 - 6$

   $\boxed{20}$

4. $\lim\limits_{x \to 8} \dfrac{\sqrt{x+1}}{x+5}$ $\quad \dfrac{\sqrt{8+1}}{8+5} = \boxed{\dfrac{3}{13}}$

5. $\lim\limits_{x \to 3} \cos \dfrac{\pi x}{2}$

   $\cos \dfrac{3\pi}{2}$

   $\boxed{0}$

6. $\lim\limits_{x \to 0} \dfrac{\dfrac{1}{x-3} + \dfrac{1}{3}}{x}$ $\quad \dfrac{\dfrac{3 + x - 3}{3(x-3)}}{x} = \dfrac{\dfrac{x}{3(x-3)}}{x}$

   $\dfrac{\cancel{x}}{3(x-3)} \cdot \dfrac{1}{\cancel{x}} = \dfrac{1}{3(x-3)}$

   $\dfrac{1}{3(0-3)} = \boxed{-\dfrac{1}{9}}$

7. $\lim\limits_{x \to 25} \dfrac{\sqrt{x} - 5}{25 - x} \dfrac{(\sqrt{x}+5)}{(\sqrt{x}+5)}$

   $\dfrac{\cancel{x - 25}}{(\cancel{25 - x})(\sqrt{x}+5)} = -\dfrac{1}{\sqrt{x}+5}$ $\quad \boxed{-\dfrac{1}{10}}$

8. $\lim\limits_{x \to 0} \dfrac{1 - \cos^2 x}{x} = \dfrac{(1-\cos x)(1+\cos x)}{x} = \dfrac{(1-\cos x)}{x} \cdot \lim(1+c$

   $\boxed{0}$

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9. $\lim_{x\to 0}\left(2+\frac{3}{x}\right) = \frac{2x+3}{x}$

$\boxed{DNE}$

10. $\lim_{x\to \frac{1}{2}^-}\frac{x^2+1}{\tan \pi x}$

$\boxed{0}$

11. $\lim_{x\to \infty}\cos\frac{1}{x}$

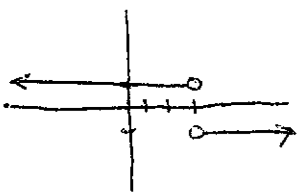
$\boxed{1}$

12. $\lim_{x\to 2}\frac{x^3-8}{x-2}$

$\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$

$4+4+4 = \boxed{12}$

13. $\lim_{x\to 3^+}\frac{|3-x|}{3-x}$



$\boxed{-1}$

14. $\lim_{x\to \infty}\frac{4x^3-5}{16x^3+2x^2+5} = \frac{\frac{4x^3}{x^3}-\frac{5}{x^3}}{\frac{16x^3}{x^3}+\frac{2x^2}{x^3}+\frac{5}{x^3}}$

$\frac{4-\frac{5}{x^3}}{16+\frac{2}{x}+\frac{5}{x^3}} \qquad \frac{4-0}{16+0+0} = \boxed{\frac{1}{4}}$

15. $\lim_{x\to \infty}\left(\frac{1}{x^2}-1\right) \qquad \frac{1-x^2}{x^2}$

$\boxed{-1}$

16. $\lim_{x\to -\infty}\frac{-3x+1}{\sqrt{x^2+x}}$

$\frac{\frac{-3x}{-x}+\frac{1}{-x}}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}} = \frac{3+\frac{1}{x}}{\sqrt{1+\frac{1}{x}}} = \frac{3}{1} = \boxed{3}$

17. $\lim_{x\to \infty^-}\frac{\sin x}{x}$

$\boxed{0}$

18. $\lim_{x\to 0^+}3\cot x \qquad \boxed{\infty}$

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19. Use the Intermediate Value Theorem to show that the function $f(x) = 2x^3 - x + 3$ has a zero in the interval [-2, -1].

$f(-2) = 2(-2)^3 - (-2) + 3$

$-16 + 5$

-11

$f(-1) = 2(-1)^3 - (-1) + 3$

$-2 + 4 = 2$

SINCE $f(c) = 0$ IS BETWEEN $f(-2)$ & $f(-1)$, THE IVT IS APPLICABLE FOR $f(x)$ ON $[-2, -1]$

20. Does the function $f(x) = \dfrac{2x^2 + 5x - 3}{x^2 - 9}$ have any discontinuities? If so, tell whether they are removable or nonremovable discontinuities.

$f(x) = \dfrac{(2x-1)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$

$f(x)$ HAS A REMOVABLE DISCONTINUITY @ $X = -3$ & AND A NON REMOVABLE @ $X = 3$

21. Find the constant $\boldsymbol{a}$ such that the function is continuous for all real numbers.

$$g(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \neq a \\ 4 & x = a \end{cases}$$

$\dfrac{x^2 - a^2}{x - a} = 4$

$\dfrac{(x+a)\cancel{(x-a)}}{\cancel{x-a}} = 4$

$x + a = 4$

$2a = 4$

$a = 2$

22. $\lim\limits_{x \to 0} \dfrac{\sin 4x}{\sin 7x} \cdot \dfrac{x}{x}$

$\dfrac{7x}{7\sin 7x} \cdot \dfrac{4\sin 4x}{4x} = \dfrac{4}{7} \approx 0.571$