Key

1) $f(x) = \frac{1}{x}$

$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x - (x+h)}{x(x+h)}}{h} = \frac{-h}{x(x+h)} \cdot \frac{1}{h} = \frac{-1}{x(x+h)} = \boxed{\frac{-1}{x^2}}$

2) $y = x \sin x$

$\frac{dy}{dx} = x \cdot \cos x + (1) \sin x$

(B) $\frac{dy}{dx} = \sin x + x \cos x$

3) $f(x) = \frac{x}{x+2}$ * Set the derivative equal to $\frac{1}{2}$

$f'(x) = \frac{(x+2)(1) - x(1)}{(x+2)^2} = \frac{2}{(x+2)^2}$

$\frac{2}{(x+2)^2} = \frac{1}{2}$ * cross multiply

$(x+2)^2 = 4$

$x+2 = \pm 2$

$x = 0, -4$

(C) $(0, 0)$ & $(-4, 2)$

4) $f(x) = x^3 - x^2 + x - 1$

$f'(x) = 3x^2 - 2x + 1$

(B) $f'(2) = 3(2)^2 - 2(2) + 1 = 9$

5) $\frac{d}{dx}\left(\sin^3(x^2)\right)$ ← THIS MEANS $\left(\sin(x^2)\right)^3$

$3(\sin(x^2))^2 \cdot \cos(x^2) \cdot 2x$

(I) $6x \sin^2(x^2)\cos(x^2)$

6) $y = 5x\sqrt{x^2+1}$

$\frac{dy}{dx} = 5x \cdot \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x + 5(x^2+1)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{5x^2}{\sqrt{x^2+1}} + 5\sqrt{x^2+1}$

$\left.\frac{dy}{dx}\right|_{x=3} = \frac{45}{\sqrt{10}} + 5\sqrt{10}$

(D) $\frac{45}{\sqrt{10}} + 5\sqrt{10}$

7) $f(x) = \sqrt{x^2+11}$ $g(x) = 3x-2$

$f'(x) = \frac{1}{2}(x^2+11)^{-\frac{1}{2}} \cdot 2x$ $g'(x) = 3$

$f'(x) = \frac{x}{\sqrt{x^2+11}}$

$f'(g(x)) \cdot g'(x) = \frac{3x-2}{\sqrt{(3x-2)^2+11}} \cdot 3$

(A) $f'(2) = \frac{1}{\sqrt{15}} \cdot 3 = \frac{21}{3\sqrt{5}}$ $\frac{1}{\sqrt{5}}$

8) $(x+2y)\frac{dy}{dx} = 2x - y$

$\frac{dy}{dx} = \frac{2x-y}{x+2y}$ $\left.\frac{dy}{dx}\right|_{(3,0)} = \frac{2(3)-0}{3+2(0)} = 2$

$\frac{d^2y}{dx^2} = \frac{(x+2y)\left(2-\frac{dy}{dx}\right) - (2x-y)\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$

(A) $\left.\frac{d^2y}{dx^2}\right|_{(3,0)} \frac{(3)(0) - (6)(5)}{9} = \frac{-30}{9} = \frac{-10}{3}$

7) DNE

10) $x^2 - xy + y^2 = 7$

$2x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{2y-2x}{x-2y}$

$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{6-2}{1-6} = \frac{4}{-5} = -\frac{4}{5}$ ← SOLVE

(L) $y - 3 = -\frac{11}{3}(x-1)$

11) $f(x) = \sin^2 x + \cos^2 x$

$f'(x) = 2\sin x \cos x - 2\cos x \sin x$

$f'(x) = 0$

12) $y = \sin(x^2 y)$

$\frac{dy}{dx} = \cos(x^2 y) \cdot \left(1 \cdot \frac{dy}{dx}\right)$

$\frac{dy}{dx} = \cos(x^2 y) - \cos(x^2 y)\frac{dy}{dx}$

$\frac{dy}{dx} + \cos(x^2 y)\frac{dy}{dx} = \cos(x^2 y)$

$\frac{dy}{dx}\left(1 + \cos(x^2 y)\right) = \cos(x^2 y)$

$\frac{dy}{dx} = \frac{\cos(x^2 y)}{1 + \cos(x^2 y)}$

13) Omit

14) $\frac{dV}{dt} = -3 \text{ cm}^3/\text{sec}$ $\frac{dr}{dt} = -0.25 \text{ cm/sec}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

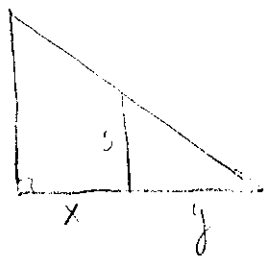
$-3 = 4\pi r^2 (-0.25)$

$-3 = \pi r^2$

$\frac{3}{\pi} = r^2$

$\sqrt{\frac{3}{\pi}} = r$

15)



$\frac{dx}{dt} = 1\frac{1}{2}$ ft/s

$\frac{dy}{dt} = ?$

$\frac{20}{x+y} = \frac{5}{y}$

$20y = 5x + 5y$

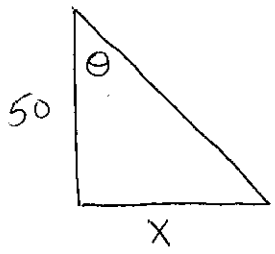
$15y = 5x$

$15\frac{dy}{dt} = 5\frac{dx}{dt}$

$15\frac{dy}{dt} = 5(4)$

$\frac{dy}{dt} = \frac{4}{3}$ ft/s

16)



$\frac{d\theta}{dt} = 2 \text{ REVS/MIN}$

$\tan\theta = \frac{x}{50}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{50} \frac{dx}{dt}$

$\sec\frac{\pi}{3} = 2$

$2^2 (2) = \frac{1}{50} \frac{dx}{dt}$

$400 = \frac{dx}{dt}$

400 METER/MIN