Average Value of a Function

How have we found Average Velocity?

How have we found Average Acceleration?

If $p(t)$, $v(t)$, and $a(t)$ represent position, velocity and acceleration defined for any time t, write an equivalent expression for each of the following integrals based on the fundamental theorem of calculus.

$\frac{1}{b-a}\int_a^b a(t)dt =$		To what is this equivalent?
$\frac{1}{b-a}\int_a^b v(t)dt =$		To what is this equivalent?

The average value of a function, $f(x)$, on an interval $[a, b]$ is defined to be:

Find the average value of the function $f(x) = x^3\sqrt{\sin^2 x}$ on the interval $1 \le x \le 3$. [Calculator]

Find the average value of the function $f(x) = 2 - 4x$ on the interval $2 \le x \le 6$. [Noncalculator]

A ski resort uses a snow machine to control the snow level on a ski slope. Over a 24-hour period the volume of snow added to the slope per hour is modeled by the equation $S(t) = 24 - t\sin^2\left(\frac{t}{14}\right)$. The rate at which the snow melts is modeled by the equation $M(t) = 10 + 8\cos\left(\frac{t}{3}\right)$. Both $S(t)$ and $M(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 24$. At time $t = 0$, the slope holds 50 cubic yards of snow.

a. Compute the total volume of snow added to the mountain over the first 6-hour period.

b. Find the value of $\int_0^6 M(t)dt$ and $\frac{1}{6}\int_0^6 M(t)dt$. Using correct units of measure, explain what each represents in the context of this problem.

c. Is the volume of snow increasing or decreasing at time $t = 4$? Justify your answer.

d. How much snow is on the slope after 5 hours? Show your work.

e. Suppose the snow machine is turned off at time $t = 10$. Write, but do not solve, an equation that could be solved to find the time $t = K$ when the snow would all be melted.

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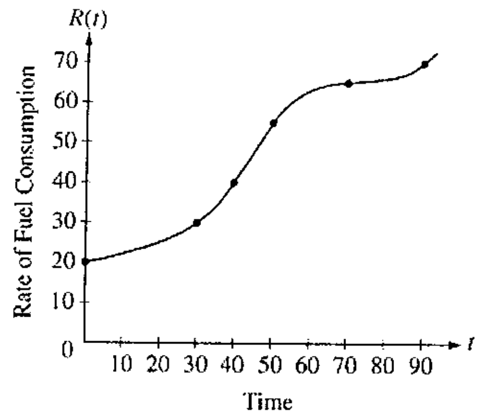
Problem #3



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

Name____Date____________________Class__________

Homework 6.7

1. Using a right Riemann sum over the given intervals, estimate $\int_5^{35} F(t)dt$

t	5	13	22	27	35
$F(t)$	44	12	13	17	22

A. 730
B. 661
C. 564
D. 474
E. 325

2. For the first six seconds of driving, a car accelerates at a rate of $a(t) = 10\sin\left(1 + \frac{t^2}{10}\right)$ meters per second^2. Which one of the following expressions represents the velocity of the car when it first begins to decelerate?

A. $\int_0^{0.775} a(t)dt$
B. $\int_0^{2.389} a(t)dt$
C. $\int_0^{1.715} a(t)dt$
D. $\int_0^{4.627} a(t)dt$
E. $\int_0^{3.830} a(t)dt$

3. The rate at which gas is flowing through a large pipeline is given in thousands of gallons per month in the chart below.

t (months)	0	3	6	9	12
$R(t)$ (1000 gallons per month)	43	62	56	60	68

Use a midpoint Riemann sum with two equal subintervals to approximate the number of gallons that pass through the pipeline in a year.

A. 594,000
B. 672,000
C. 732,000
D. 744,000
E. 1,068,000

4. Let f be a continuous function on the closed interval [1, 11]. If the values of f are given below at three points, use a trapezoidal approximation to find $\int_1^{11} f(x)dx$ using two subintervals.

A. 165
B. 172
C. 190.5
D. 40
E. 80

x	1	9	11
$f(x)$	23	14	10

5. If $\int_a^b f(x)dx = 2a - 3b$, then $\int_a^b [f(x) + 3]dx =$

A. $2a - 3b + 3$
B. $3b - 3a$
C. $-a$
D. $5a - 6b$
E. $a - 6b$

Use the table below to answer questions 6 and 7. Suppose the function $f(x)$ is a continuous function and f is the derivative of $F(x)$.

x	0	1	2	3
$f(x)$	–1	0	1	–2
$F(x)$	4	3	A	8

6. What is $\int_1^3 f(x)dx$?

A. 5
B. 8
C. 4
D. 19
E. Cannot be determined

7. If the area under the curve of $f(x)$ on the interval $0 \le x \le 2$ is equal to the area under the curve $f(x)$ on the interval $2 \le x \le 3$, then what is the value of A?

A. 4
B. 2
C. 5.5
D. 6
E. Cannot be determined

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Problem #1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

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Problem #2

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.