Answers and Answer Explanations

Using the table below, score your test. Determine how many questions you answered correctly and how many you answered incorrectly. Additional information about scoring is at the end of the Practice Test.

1. B	2. A	3. D	4. B	5. D
6. B	7. D	8. A	9. C	10. B
11. A	12. C	13. B	14. D	15. C
16. C	17. C	18. D	19. D	20. D
21. C	22. D	23. C	24. B	25. B
26. B	27. B	28. C	29. A	30. C
31. B	32. A	33. D	34. D	35. A
36. A	37. B	38. A	39. D	40. B
41. C	42. B	43. C	44. C	45. C

1. ANSWER: **(B)** $\int \frac{x-3}{x}\,dx = \int 1 - \frac{3}{x}\,dx = x - 3\ln x + C$ (*Calculus for AP* 1st ed. pages 280–286; EU 3.3; LO 3.3B; EK 3.3B5; MPAC 3,5)

2. ANSWER: **(A)** Rewrite the limit into factored form, simplify, and substitute $x = -1$.
$$\lim_{x\to -1}\frac{x^2-3x-4}{x^2-1} = \lim_{x\to -1}\frac{(x-4)(x+1)}{(x-1)(x+1)} = \lim_{x\to -1}\frac{(x-4)}{(x-1)} = \frac{-5}{-2} = \frac{5}{2}$$
(*Calculus for AP* 1st ed. pages 76–83; EU 1.1; LO 1.1C; EK 1.1C2; MPAC 3,5)

3. ANSWER: **(D)** $f'(x) = 3 + 10\sin 2x$, and $f'(0) = 3$. The slope of the normal line is $-1/3$. For $f(0) = -5$, the normal is $y + 5 = (-1/3)(x - 0)$, or $y = (-1/3)x - 5$. (*Calculus for AP* 1st ed. pages 158–167; EU 2.3; LO 2.3B; EK 2.3B1; MPAC 2,3)

4. ANSWER: **(B)** A particle speeds up in any interval when the velocity and acceleration have the same sign. Take the first and second derivative to find velocity and acceleration. Set $v(t) = 4t^3 - 36t = 4t(t^2-9) = 0$ and $a(t) = 12t^2 - 36 = 12(t^2-3) = 0$ and use the interval chart to analyze the motion of the particle.

	$0 < t < \sqrt{3}$	$\sqrt{3} < t < 3$	$3 < t < 5$
$v(t)$	negative	negative	positive
$a(t)$	negative	positive	positive
Particle	speeding up	slowing down	speeding up

(*Calculus for AP* 1st ed. pages 147–153; EU 2.3; LO 2.3C; EK 2.3C1; MPAC 2,3)

5. **ANSWER: (D)** Find $f'(x)$ and $f''(x)$ and evaluate each limit.

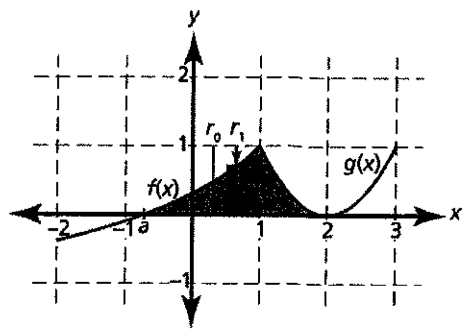
Case I: $f(x) = e^x \sin x$ and $\lim_{x\to 0} e^x \sin x = e^0(0) = 1(0) = 0$

Case II: $f'(x) = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$ and $\lim_{x\to 0} f'(x) = e^0(0+1) = 1$

Case III: $f''(x) = e^x(\sin x + \cos x) + e^x(\cos x - \sin x) = 2e^x \cos x$ and $\lim_{x\to 0} f'(x) = 2e^0(1) = 2$

Therefore none of the statements is false. (*Calculus for AP* 1st ed. pages 147–153, 163; EU 1.1,2.1; LO 1.1C,2.1C; EK 1.1C1,2.1C3; MPAC 2,3,5)

6. **ANSWER: (B)** Using the Washer Method,
$V = \pi \int_a^1 \left[1^2 - (1-f(x))^2\right] dx +$
$\pi \int_1^2 \left[1^2 - (1-g(x))^2\right] dx$, where $r_o = 1$ and $r_i = 1 - y$.



(*Calculus for AP* 1st ed. pages 421–422; EU 3.4; LO 3.4D; EK 3.4D2; MPAC 2,4)

7. **ANSWER: (D)**

$$\begin{aligned} f'(x) &= 3(3+2x-x^2)^2(2-2x) \\ &= 3(3-x)^2(1+x)^2 \cdot 2(1-x) \\ &= 0 \end{aligned}$$

Evaluate $f(x)$ at the critical points and the end points of the closed interval.

x	−2	−1	1	3
$f(x)$	−125	0	64	0
extrema	absolute minimum		absolute maximum	

Therefore, $M = -125$, $m = 64$, and $|M + m| = 61$. (*Calculus for AP* 1st ed. pages 212–216; EU 2.3; LO 2.3C; EK 2.3C3; MPAC 1,2,3)

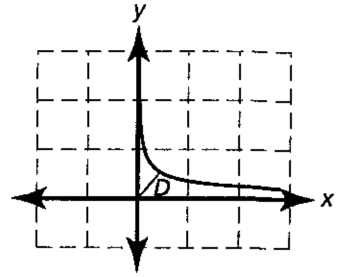
8. **ANSWER: (A)** If the slope of the curve is $\left(3+\frac{1}{x}\right)y$, then $\frac{dy}{dx} = \left(3+\frac{1}{x}\right)y$. Separating the variables yields $\int \frac{dy}{y} = \int 3 + \frac{1}{x}\, dx$, for $x > 0$. Therefore, $\ln|y| = 3x + \ln x + C_1$ and $y = e^{3x+\ln x + C_1} = Ce^{3x}e^{\ln x} = Cxe^{3x}$. Using the initial condition (1, 2), it follows that $2 = Ce^3 \Rightarrow C = \frac{2}{e^3}$. By substitution and the rules for exponents, we simplify and get $y = \frac{2}{e^3}xe^{3x} = 2xe^{3x-3}$. (*Calculus for*

AP 1st ed. pages 385–390; EU 3.5; LO 3.5A; EK 3.5A1,3.5A2; MPAC 2,3)

9. **ANSWER: (C)** Evaluate the sum using trapezoids.

$\frac{1}{2}[2(10+20)+3(20+50)+1(50+80)]=\frac{1}{2}(400)=200$ (*Calculus for AP* 1st ed. pages 302–306; EU 3.2; LO 3.2B; EK 3.2B1,3.2B2; MPAC 2,4)

10. **ANSWER: (B)** Let the point of the curve be given by $P\left(x,\frac{4}{\sqrt{x}}\right)$. Let the square of the distance between the origin and P be defined by $D(x)=(x-0)^2+\left(\frac{4}{\sqrt{x}}-0\right)^2=x^2+\frac{16}{x}$. Therefore, $D'(x)=2x-\frac{16}{x^2}=0$ and then $x=2$. With $D''(x)=2+\frac{32}{x^3}$ and $D''(2)>0$, then $x=2$ is the x coordinate of the point on the curve that is closest to the origin. And the y coordinate is $\frac{4}{\sqrt{2}}=2\sqrt{2}$.



(*Calculus for AP* 1st ed. pages 257–261; EU 2.3; LO 2.3C3; EK 2.3C3; MPAC 1,2,3)

11. **ANSWER: (A)** Since the degree of the numerator is equal to the degree of the denominator $\left(\sqrt{x^2}=x\right)$, divide each term in the numerator and denominator by x. The resulting limit can then be evaluated: $\lim_{x\to\infty}\frac{\frac{3}{x}-\sqrt{\frac{x^2}{x^2}-\frac{1}{x^2}}}{\frac{2x}{x}+\frac{5}{x}}=\lim_{x\to\infty}\frac{\frac{3}{x}-\sqrt{1-\frac{1}{x^2}}}{2+\frac{5}{x}}=\frac{0-\sqrt{1-0}}{2+0}=-\frac{1}{2}$.

(*Calculus for AP* 1st ed. pages 108–114; EU 1.1; LO 1.1C; EK 1.1C2,1.1C3; MPAC 2,3,5)

12. **ANSWER: (C)** Differentiating implicitly with respect to t, $\sec^2 y\left(\frac{dy}{dt}\right)+3x^2\left(\frac{dx}{dt}\right)=2y\left(\frac{dy}{dt}\right)$. Substituting, $\sec^2 0\left(\frac{dy}{dt}\right)+3(1)^2(-2)=2(0)\left(\frac{dy}{dt}\right)\Rightarrow\frac{dy}{dt}-6=0$; therefore $\frac{dy}{dt}=6$.

(*Calculus for AP* 1st ed. pages 189–193; EU 2.3; LO 2.3C; EK 2.3C2; MPAC 2,3,5)

13. **ANSWER: (B)** $v_{avg} = \frac{1}{2-0}\int_0^2 3t^2 - e^t\,dt$ becomes

$$\frac{1}{2}\left(t^3 - e^t\right)\Big|_0^2 = \frac{1}{2}\left[\left(2^3 - e^2\right) - \left(0 - e^0\right)\right] = \frac{1}{2}\left(8 - e^2 + 1\right) = \frac{9-e^2}{2}.$$

(*Calculus for AP* 1st ed. pages 317–325; EU 2.1; LO 2.1A; EK 2.1A1; MPAC 2,3,5)

14. **ANSWER: (D)** $f'(x) = \begin{cases} 4kx - 1, & x > 3 \\ 3x^2 + c, & x \le 3 \end{cases}$. If $f(x)$ is differentiable at $x = 3$, then $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$ and $\lim_{x\to 3^-} f'(x) = \lim_{x\to 3^+} f'(x)$. The solution of the system $18k - 3 = 27 + 3c$ and $12k - 1 = 27 + c$, is $k = 3$ and $c = 8$. Thus $(k + c) = 11$. (*Calculus for AP* 1st ed. pages 124–130; EU 1.2,2.2; LO 1.2A,2.2B; EK 1.2A1,2.2B2; MPAC 1,2,3)

15. **ANSWER: (C)** $x(8) = x(0) + \int_0^8 v(t)\,dt$

$$\begin{aligned} x(8) &= -4 + \int_0^8 \left(t - t^{\frac{1}{3}}\right)dt \\ &= -4 + \left(\frac{t^2}{2} - \frac{3}{4}t^{\frac{4}{3}}\right)\Bigg|_0^8 \\ &= -4 + \left[\frac{64}{2} - \frac{3}{4}\left(8^{\frac{4}{3}}\right)\right] \\ &= -4 + 32 - 12 = 16 \end{aligned}$$

Therefore, the particle is 16 cm to the right of the origin. (*Calculus for AP* 1st ed. pages 280–286; EU 3.3; LO 3.3B; EK 3.3B2; MPAC 2,3)

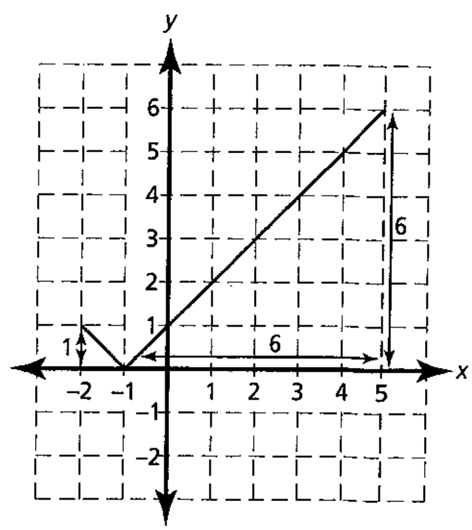
16. **ANSWER: (C)** Solving the problem geometrically, the integral given equals the sum of the areas of two right triangles. For $f(x) = |x+1|$, the x-intercept is -1, and the dimensions of the triangles are labeled on the diagram.

An alternate method would be to express $f(x)$ as a piecewise function and perform the integration. Thus, $f(x) = \begin{cases} -x - 1, & x < -1 \\ x + 1, & x \ge -1 \end{cases}$ and the area $= \int_{-2}^{-1}(-x-1)\,dx + \int_{-1}^{5}(x+1)\,dx$. Then,

$$\left(-\frac{x^2}{2} - x\right)\Bigg|_{-2}^{-1} + \left(\frac{x^2}{2} + x\right)\Bigg|_{-1}^{5}$$

$$= \left[-\frac{1}{2} + 1 - (-2 + 2)\right] + \left[\frac{25}{2} + 5 - \left(\frac{1}{2} - 1\right)\right]$$

$$\Rightarrow \frac{1}{2} + 12 + 6 = 18.5.$$



Area $= \frac{1}{2}(1)(1) + \frac{1}{2}(6)(6) = 18.5$

(*Calculus for AP* 1st ed. pages 302–306; EU 3.2; LO 3.2C; EK 3.2C1; MPAC 4,5)

17. ANSWER: (C) $\int\left(\frac{x+2}{x^2+1}\right)dx = \int\left(\frac{x}{x^2+1}\right)dx + \int\left(\frac{2}{x^2+1}\right)dx$. For the first integral, let $u = x^2+1$, and $du = 2xdx$. After substitution, $\frac{1}{2}\int\frac{du}{u} + \int\frac{2}{x^2+1}dx = \frac{1}{2}\ln u + 2\tan^{-1}x + C$ or $\frac{1}{2}\ln(x^2+1) + 2\tan^{-1}x + C$.
(*Calculus for AP* 1st ed. pages 332–334, 354–355; EU 3.3; LO 3.3B; EK 3.3B5; MPAC 2,3)

18. ANSWER: (D) The equation of the tangent at $x = 3$ is given by $y - 5 = -2(x - 3)$. When $x = 2.98$, $y = 5 + (-2)(0-.02) = 5.04$. (*Calculus for AP* 1st ed. pages 267–271; EU 2.3; LO 2.3B; EK 2.3B2; MPAC 2,3)

19. ANSWER: (D) The slope field for $\frac{dy}{dx} = \frac{e^x}{y}$ will have vertical segments at all points on the x-axis because $y = 0$. Those points above the x-axis will have segments with positive slopes, and those below will have negative slopes, as e^x is always positive. (*Calculus for AP* 1st ed. pages 368–371; EU 2.3; LO 2.3F; EK 2.3F1; MPAC 4)

20. ANSWER: (D) Although the function may be discontinuous at $x = 10$, the limit does exist if $\lim_{x\to 10^-} g(x) = \lim_{x\to 10^+} g(x)$. Therefore, $\lim_{x\to 10^-} e^{\left(\frac{x}{10}+C\right)} = \lim_{x\to 10^+}(\log(x)+1)$. Then, $e^{\left(\frac{10}{10}+C\right)} = \log(10)+1 \Rightarrow e^{1+C} = 1+1 \Rightarrow e^{1+C} = 2$. Finally, $1 + C = \ln 2$, and $C = \ln 2 - 1$. (*Calculus for AP* 1st ed. pages 65–70; EU 1.1; LO 1.1A; EK 1.1A2; MPAC 2,3)

21. ANSWER: (C) For $y = 4^{x^2}$, $y' = 4^{x^2}(\ln 4)(2x)$ and $y' = 4(\ln 4)(2) = 8\ln 4$. (*Calculus for AP* 1st ed. pages 166–167; EU 2.1; LO 2.1C; EK 2.1C2; MPAC 2,3)

22. ANSWER: (D) By the Second Fundamental Theorem, $g(x) = 3 + \left[1+(x^2)^2\right](2x) = 3 + (1+x^4)(2x)$. Then, $g(2) = 3 + (1 + 16)(4) = 71$.(*Calculus for AP* 1st ed. pages 317–325; EU 3.3; LO 3.3A; EK 3.3A2; MPAC 3,5)

23. ANSWER: (C) There is no root for $f(x)$, because $e^x + 1 \neq 0$. There is a vertical asymptote at $x = 0$ because $e^0 - 1 = 0$. And there is no y-intercept at $x = 0$ because $f(x)$ is not defined at $x = 0$. The only true statement is found by taking the derivative and analyzing it: $f'(x) = \frac{e^x(e^x-1) - e^x(1+e^x)}{(e^x-1)^2} = \frac{-2e^x}{(e^x-1)^2}$. There are no critical points for $f(x)$ and $f'(x)$ is always less than 0, so $f(x)$ is decreasing for all x, provided $x \neq 0$. (*Calculus for AP* 1st ed. pages 163–165; EU 1.1,2.2; LO 1.1D,2.2A; EK 1.1D1,2.2A1; MPAC 2,3)

24. ANSWER: (B) The particle moves to the left until time E because the velocity is less than 0 and is therefore farthest to the left of its

starting position. (*Calculus for AP* 1st ed. pages 227–232; EU 3.4; LO 3.4A; EK 3.4A2; MPAC 4)

25. **ANSWER: (B)** Differentiating implicitly yields
$(\cos xy)\left[y + x\dfrac{dy}{dx}\right] = 1 + \dfrac{dy}{dx}$. Therefore, $\dfrac{dy}{dx} = \dfrac{y\cos xy - 1}{1 - x\cos xy}$.

(*Calculus for AP* 1st ed. pages 173–177; EU 2.1; LO 2.1C; EK 2.1C5; MPAC 2,3)

26. **ANSWER: (B)** $g(0) = \int_{-4}^{0} f(t)\,dt = \frac{1}{2}(3)(2) - \frac{1}{2}(1)(2) = 2$

(*Calculus for AP* 1st ed. pages 317–325; EU 3.2; LO 3.2C; EK 3.2C1; MPAC 2,4)

27. **ANSWER: (B)** $g(3) = \int_{-4}^{3} f(t)\,dt = \frac{1}{2}(3)(2) - \frac{1}{2}(2)(4+2) = 3 - 6 = -3$, and by the Second Fundamental Theorem, $g'(3) = f(3) = 0$, so the tangent is $y = -3$. (*Calculus for AP* 1st ed. pages 317–325; EU 2.3; LO 2.3B; EK 2.3B1; MPAC 2,3,4)

28. **ANSWER: (C)** A relative maximum or minimum occurs where the first derivative of a function changes sign. Since $g(x) = \int_{2}^{x} f(t)\,dt$ and $g'(x) = f(x)$, analyze the behavior of $g(x)$ using an interval chart.

	$-4 < x < -1$	$x = -1$	$-1 < x < 3$	$x = 3$	$3 < x < 4$
$g'(x) = f(x)$	positive	0	negative	0	positive
$g(x)$	increasing	rel.max	decreasing	rel.min	increasing

Statement C is therefore false, as there is no relative maximum at $x = -2$. (*Calculus for AP* 1st ed. pages 317–325; EU 2.2; LO 2.2A; EK 2.2A1; MPAC 2,3,4)

29. **ANSWER: (A)** Using L'Hopital's Rule for the indeterminate form

$$\frac{0}{0},\ \lim_{x\to 0}\frac{\sin 6x}{x^2 - 3x} = \lim_{x\to 0}\frac{\frac{d}{dx}(\sin 6x)}{\frac{d}{dx}\left(x^2 - 3x\right)} = \lim_{x\to 0}\frac{6\cos 6x}{2x - 3} = \frac{6\cdot 1}{0-3} = \frac{6}{-3} = -2$$

(*Calculus for AP* 1st ed. pages 504–510; EU 1.1; LO 1.1A; EK 1.1C3; MPAC 1,2,3,4)

30. **ANSWER: (C)** Using L'Hopital's Rule for the indeterminate form 0/0 and the Fundamental Theorem of Calculus

$$\lim_{x\to 1}\frac{\int_{2}^{4-2x}\sqrt{t+7}\,dt}{x-1} = \lim_{x\to 1}\frac{\frac{d}{dx}\left(\int_{2}^{4-2x}\sqrt{t+7}\,dt\right)}{\frac{d}{dx}(x-1)} = \lim_{x\to 1}\frac{-2\sqrt{4-2x+7}}{1} = \frac{-2\sqrt{9}}{1} = -6$$

(*Calculus for AP* 1st ed. pages 504–510; EU 1.1; LO 1.1A; EK 1.1C3,3.3A2; MPAC 1,2,3,4)

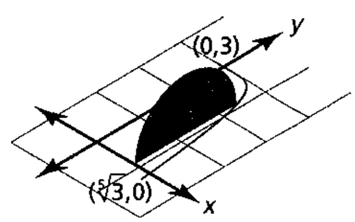
31. **ANSWER: (B)** Finding the intersection point (–0.8834, 1.2181), the area is determined by evaluating the integral
$\int_{-0.8834}^{0} -\tan x - 2x^4\,dx \approx 0.240.$
(*Calculus for AP* 1st ed. pages 408–413; EU 3.4; LO 3.4D; EK 3.4D1; MPAC 3,4)

32. **ANSWER: (A)** The average rate of change is equivalent to the slope of the secant on the given interval. Thus,
$$\frac{f(-1)-f(-4)}{-1-(-4)} = \frac{e^{-1} - \frac{e^{-\frac{1}{4}}}{16}}{3} \approx 0.106.$$
(*Calculus for AP* 1st ed. pages 220–223; EU 3.4; LO 3.4B; EK 3.4B1; MPAC 2,3)

33. **ANSWER: (D)** $u = \cos 2x \Rightarrow du = -2\sin 2x\,dx;$ and for
$x = 0 \Rightarrow u = 1$ and for $x = \frac{\pi}{2} \Rightarrow u = -1.$
Then by substitution $\int_{0}^{\frac{\pi}{2}} \sin 2x\, e^{\cos 2x}\,dx = -\frac{1}{2}\int_{1}^{-1} e^{u}\,du \Rightarrow \frac{1}{2}\int_{-1}^{1} e^{u}\,du.$
(*Calculus for AP* 1st ed. pages 332–334; EU 3.3; LO 3.3B; EK 3.3B5; MPAC 3,5)

34. **ANSWER: (D)** Area = $\int_{k}^{k+1} \frac{1}{x}\,dx = \ln(k+1) - \ln k = 0.125$. Then
$\ln\left(\frac{k+1}{k}\right) = 0.125 \Rightarrow e^{0.125} = \frac{k+1}{k}$ and $k \approx 7.510.$
(*Calculus for AP* 1st ed. pages 345–349; EU 3.4; LO 3.4D; EK 3.4D1; MPAC 2,3)

35. **ANSWER: (A)** The function crosses the x-axis at $x = 3^{\frac{1}{5}} \approx 1.2457$. The radius of each semicircle = $\frac{3-x^5}{2}$
and the area of each cross section is $\frac{\pi}{2}\left(\frac{3-x^5}{2}\right)^2$, so
the volume is given by $\frac{\pi}{8}\int_{0}^{1.2457} (3-x^5)^2\,dx \approx 3.335.$



(*Calculus for AP* 1st ed. pages 423–424; EU 3.4; LO 3.4D; EK 3.4D2; MPAC 2,3)

36. **ANSWER: (A)** The amount left in the bottle is given by
$32 - \int_{0}^{5} Y(t)\,dt \approx 32 - 5.063 = 26.937$ ounces.

(*Calculus for AP* 1st ed. pages 317–325; EU 3.3; LO 3.3B; EK 3.3B2; MPAC 2,3)

37. **ANSWER: (B)** By the Mean Value Theorem $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ for $a \le c \le 2$, we have $f'(1.720) = \dfrac{f(2)-f(a)}{2-a} \Rightarrow 8.875 = \dfrac{8-a^3}{2-a}$. The solution to this equation is $a = 1.424$. (*Calculus for AP* 1st ed. pages 220–223; EU 1.2,2.4; LO 1.2B,2.4A; EK 1.2B1,2.4A1; MPAC 2,3)

38. **ANSWER: (A)** If $f(x)$ is integrable on $p \le x \le q$, then $\int_q^p f(x)dx = -\int_q^p f(x)dx$. The given information yields the following system: $P+Q+R=-7$, $Q+R=-2$, and $-(P+Q)=17$. Solving the system, $P=-5, Q=-12$, and $R=10$, and then $\int_b^c f(x)\,dx = Q = -12$. (*Calculus for AP* 1st ed. pages 317–325; EU 3.2; LO 3.2C; EK 3.2C1; MPAC 2,4,5)

39. **ANSWER: (D)** For $g(5) = 3$, then $f(3) = 5$ and $g'(5) = \dfrac{1}{f'(3)} = \dfrac{1}{2}$. Differentiating, $h'(x) = g(x) + xg'(x) \Rightarrow$

$$h'(5) = g(5) + 5 \cdot g'(5) = 3 + 5\left(\frac{1}{2}\right) = 5.5.$$

(*Calculus for AP* 1st ed. pages 182–183; EU 2.1; LO 2.1C; EK 2.1C6; MPAC 2,4,5)

40. **ANSWER: (B)** $V = \dfrac{1}{3}\pi r^2 h = \dfrac{\pi}{3}\left(\dfrac{h}{2}\right)^2 h = \dfrac{\pi h^3}{12} \Rightarrow \dfrac{dV}{dt} = \dfrac{\pi h^2}{4}\left(\dfrac{dh}{dt}\right)$. If $C = 120\pi = 2\pi r, \Rightarrow r = 60$ and $h = 120$. Substituting, $30\pi = \dfrac{\pi}{4}120^2\left(\dfrac{dh}{dt}\right) \Rightarrow \dfrac{dh}{dt} = \dfrac{1}{120}$ cm/sec. (*Calculus for AP* 1st ed. pages 189–193; EU 2.3; LO 2.3C; EK 2.3C2; MPAC 2,3)

41. **ANSWER: (C)** By the Quotient and Chain Rules,

$$h'(x) = \frac{g'(2x)\cdot 2\cdot f(x) - f'(x)g(2x)}{(f(x))^2}$$ and

$$h'(1) = \frac{g'(2)\cdot 2\cdot f(1) - f'(1)\cdot g'(2)}{(f(1))^2} = \frac{-1(2)(4)-(1)(-1)}{16} = -\frac{7}{16}.$$

(*Calculus for AP* 1st ed. pages 147–153; EU 2.1; LO 2.1C; EK 2.1C3,2.1C4; MPAC 2,3,4)

42. **ANSWER: (B)** The total number of fires for the first quarter of the year is determined by $\int_0^{91.25}\left[4\cos\left(\dfrac{t}{58}-2\right)+6\right]dt \approx 660$. (*Calculus for AP* 1st ed. pages 317–325; EU 3.4; LO 3.4A; EK 3.4A2; MPAC 2,3)

43. **ANSWER: (C)** The SLOPE of the graph of the derivative of a function must change signs in order for the graph of the original function to have a point of inflection. Thus only $h(x)$ has at least one point of inflection as the graph of $h'(x)$ changes SLOPE from positive to

negative, back to positive and then finally negative, showing three points of inflection in the interval $-3 < x < 2$. (*Calculus for AP* 1st ed. pages 237–241; EU 2.2; LO 2.2A; EK 2.2A1,2.2A3; MPAC 2,4)

44. ANSWER: (C) Graph $P'(t)$ on the interval $0 \le t \le 10$ and observe that the most minimum value of $P'(t)$ occurs at $t = 9.491$ with $P'(9.491) \approx -1.301$. Finally, $P(9.491) \approx \$19.91$. (*Calculus for AP* 1st ed. pages 237–241, 317–325; EU 2.2,2.3; LO 2.2A,2.3C; EK 2.2A1,2.3C3; MPAC 2,3)

45. ANSWER: (C) $g(x)$ is increasing when $g'(x) > 0$, and concave up when the slope of $g'(x)$, or $g''(x)$, is also > 0.

	$a<x<b$	$b<x<c$	$c<x<d$	$d<x<e$	$e<x<f$	$f<x<h$
$g'(x)$	positive	positive	negative	negative	positive	positive
$g''(x)$	positive	negative	negative	positive	positive	negative
Characteristics of $g(x)$	increasing	increasing	decreasing	decreasing	increasing	increasing
	concave up	concave down	concave down	concave down	concave up	concave down

Therefore $g(x)$ is increasing and concave up for the intervals $a < x < b$ and $e < x < f$. (*Calculus for AP* 1st ed. pages 227–232, 237–241; EU 2.2; LO 2.2A; EK 2.2A1,2.2A3; MPAC 2,4)

FREE-RESPONSE QUESTIONS

1. The velocity of a particle moving on the y-axis is given by $v(t) = \dfrac{e^{t-2}}{2t^2+3} - 4$

On the interval $1 \le t \le 11$. At time t=1, the particle is 3 units above the origin.

(a) During what time interval is the particle moving down?

(b) What is the position of the particle when it is farthest south of the origin?

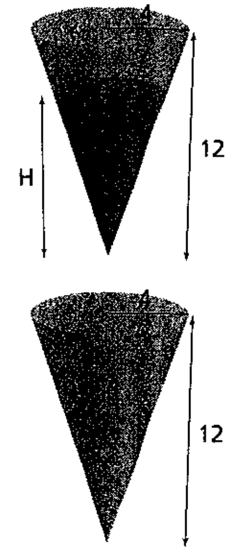
(c) At what time is the particle's average velocity twice the instantaneous velocity on $2 \le t \le 8$?

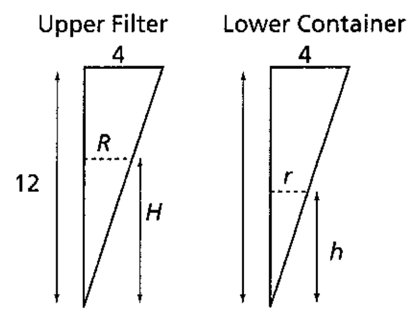
(d) Find the total distance traveled by the particle during the time where $1 \le t \le 11$.

	Solution	Possible points
(a)	The particle is moving down when $v(t) < 0$. Therefore, the particle is moving to the left in the interval $1 \le t \le 8.344$	2: $\begin{cases} 1: & v(t)<0 \\ 1: & \text{answer} \end{cases}$
(b)	$\int_1^{8.344} v(t)dt = y(8.344) - y(1)$ $-23.772 = y(8.344) - 3 = -20.772$ The particle is 20.772 units south of the origin.	3: $\begin{cases} 1: & \text{uses the FTC} \\ 1: & \text{integral and initial value} \\ 1: & \text{answer} \end{cases}$
(c)	$v_{av} = \dfrac{1}{8-2}\int_2^8 v(t)dt = -3.281$ $2v(t) = -3.281$ $t = 7.645$	2: $\begin{cases} 1: & \text{integral and constant} \\ 1: & \text{answer} \end{cases}$
(d)	Total distance = $\int_1^{11} \lvert v(t)dt \rvert = 49.302$	2: $\begin{cases} 1: & \text{integral on } \lvert v(t) \rvert \\ 1: & \text{answer} \end{cases}$

1. (a) (*Calculus for AP* 1st ed. pages 227–232; EU 2.3; LO 2.3C; EK 2.3C1; MPAC 2,3)
1. (b), (c), (d) (*Calculus for AP* 1st ed. pages 317–325; EU 2.3,2.4,3.4; LO 2.3C,2.4A,3.4C; EK 2.3C1,2.4A1,3.4C1; MPAC 2,3)

2. A solution is draining through a conical filter into an identical conical container as shown in the diagram to the right. The solution drips from the upper filter into the lower container at a rate of π cm³/sec $\left(V_{cone} = \frac{\pi}{3} r^2 h\right)$.
   (a) How fast is the level in the upper filter dropping when the solution level in the upper filter is at 6 cm?
   (b) If the conical filter is initially full, what is the level of the solution in the lower level when the solution in the level in the upper filter is at 6 cm and how fast is the level in the lower filter rising?
   (c) How fast is the surface area of the solution in the lower filter increasing when the volume in the upper filter equals the volume in the lower container?

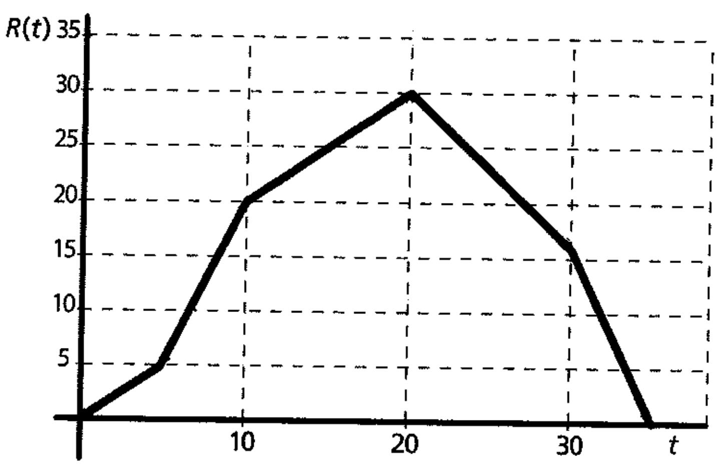


	Solution	Possible points
(a)	By similar triangles for both cones:  $\frac{4}{12} = \frac{1}{3} = \frac{R}{H} \Rightarrow R = \frac{1}{3}H$ and $V_{cone} = \frac{\pi}{3} R^2 H \Rightarrow \frac{\pi}{27} H^3$. Then $\frac{dV}{dt} = \frac{\pi}{9} H^2 \left(\frac{dH}{dt}\right) \Rightarrow -\pi = \frac{\pi}{9} 6^2 \left(\frac{dH}{dt}\right)$ $\Rightarrow \frac{dH}{dt} = -\frac{1}{4}$ cm/sec.	3: { 1: $R = \frac{1}{3}H$ 1: $\frac{dV}{dt}$ 1: answer
(b)	When $H = 6$, the volume left in the upper filter is $V_{upper} = \frac{\pi}{27} 6^3 = 8\pi$ cm^3 and the volume in the lower container is $V_{lower} = \frac{\pi}{3} 4^2(12) - 8\pi = 64\pi - 8\pi = 56\pi$ cm^3. Solving for h in the lower container, $56\pi = \frac{\pi}{27} h^3 \Rightarrow h = 6\sqrt[3]{7} \approx 11.4776$. Since $\frac{dV}{dt} = \frac{\pi}{9} h^2 \left(\frac{dh}{dt}\right) \Rightarrow \pi = \frac{\pi}{9}(11.4776)^2 \left(\frac{dh}{dt}\right)$ and $\frac{dh}{dt} \approx 0.068$ cm/sec.	3: { 1: V_{lower} 1: h_{lower} 1: answer

	Solution	Possible points
(c)	$V = \frac{\pi}{3}(4)^2(12) = 64\pi$ Half full: $V = 32\pi = \frac{\pi}{27}h^3 \Rightarrow h = 6\sqrt[3]{4} \approx 9.5244$ $\frac{dV}{dt} = \frac{\pi}{9}h^2\left(\frac{dh}{dt}\right) \Rightarrow \pi = \frac{\pi}{9}(9.5244)^2\left(\frac{dh}{dt}\right)$ and $\frac{dh}{dt} \approx 0.099$ cm/sec and $r = \frac{1}{3}h$, then $r \approx 3.175$ and $\frac{dr}{dt} = \frac{1}{3}\frac{dh}{dt} \Rightarrow 0.033$ cm/sec. $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$ and $\frac{dA}{dt} \approx 2\pi(3.175)^2(0.033) \approx 0.658$ cm²/sec.	3: $\begin{cases} 1: & h \\ 1: & \frac{dh}{dt} \\ 1: & \frac{dA}{dt} \end{cases}$

2. (a), (b), (c) (*Calculus for AP* 1st ed. pages 189–193; EU 2.3; LO 2.3C; EK 2.3C2; MPAC 2,3)

3. A 1,000 gallon tank is initially empty. A valve is opened and water begins to flow into the tank at a variable rate, $R(t)$, in gallons per minute, where t is the number of minutes since the valve was opened. The rate is recorded in the chart and graph below.



t min	$R(t)$ gal / min
0	0
5	5
10	20
20	30
30	16
35	0

(a) Approximate the volume of water that flows into the tank for the time interval $0 \le t \le 35$ using a right Riemann sum for the five unequal sub-intervals indicated by the data in the chart. Indicate units of measure.

(b) Approximate the value of $\frac{1}{20}\int_{10}^{30} R(t)dt$ using the trapezoidal rule. Indicate units of measure with your answer. Interpret the meaning of your answer in the context of the problem.

(c) At time $t = 20$ minutes, a drain is opened at the bottom of the tank and water leaves the tank at a rate of $(t + 10)$ gallons per minute. Set up, but do not evaluate, an integral expression in terms of a single independent variable that represents the volume of the water in the tank at time $t = 35$ minutes.

	Solution	Possible points
(a)	Using the data from the chart, 5(5) + 5(20) + 10(30) + 10(16) + 5(0) = 585 gal.	2: {<−1> entry error
(b)	$\frac{1}{20}\int_{10}^{30} R(t)dt$ is the average rate that water enters the tank over the 20 minute period $10 \le t \le 30$ minutes. $\frac{1}{20}\int_{10}^{30} R(t)dt$ $\approx \frac{1}{20}\left(\frac{30-10}{2(2)}\right)[R(10)+(2)R(20)+R(30)]$ $= \frac{1}{20}\left(\frac{20}{4}\right)[20+2(30)+16]$ $\Rightarrow \frac{1}{4}(96) = 24$ gal/min	3: $\begin{cases} 2: \text{ answer} \\ \quad <-1> \text{ entry error} \\ 1: \text{ interpretation} \end{cases}$
(c)	$\left[\int_0^{35} Q(t)dt\right]-\left[\int_{20}^{35}(t+10)dt\right]$	3: $\begin{cases} 1: \text{ integrand and interval for } Q(t) \\ 1: \text{ integrand and interval for } (t+10) \\ 1: \text{ correct order of functions} \end{cases}$ 1: units in parts a, and b.

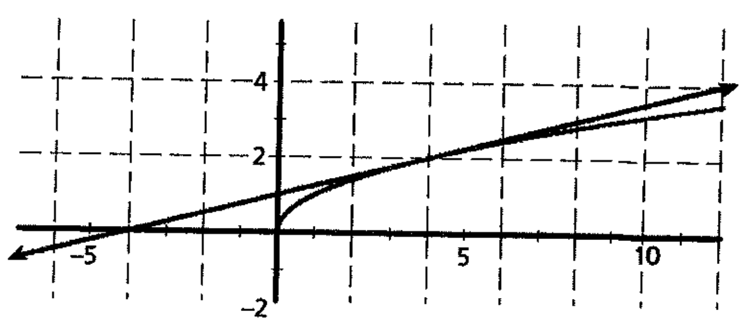
3. (a) (*Calculus for AP* 1st ed. pages 302–306; EU 3.2; LO 3.2B; EK 3.2B2; MPAC 2,3,6)
3. (b) (*Calculus for AP* 1st ed. pages 317–325; EU 3.2,3.4; LO 3.2B,3.4B; EK 3.2B2,3.4B2; MPAC 2,3,6)
3. (c) (*Calculus for AP* 1st ed. pages 220–223; EU 3.4; LO 3.4E; EK 3.4E1; MPAC 2,3,6)

4. Consider the curve defined by $x^2 + 4xy + y^2 = -12$.
   (a) Find $\frac{dy}{dx}$ in terms of x and y.
   (b) Find the equations of all horizontal tangents.
   (c) Find the equation of the tangent at the point (–4, 14).
   (d) If $\frac{dy}{dt} = -\frac{1}{2}$ at the point (–4, 14), find $\frac{dx}{dt}$.
   (e) Use the tangent in part c to estimate the value of k for the point (–4.01, k) on the curve.

	Solution	Possible points
(a)	$2x+4y+4x\left(\frac{dy}{dx}\right)+2y\left(\frac{dy}{dx}\right)=0$ $\frac{dy}{dx}=\frac{-(x+2y)}{2x+y}$	2: $\begin{cases}1: & \text{implicit differentiation} \\ & <-1> \text{each error} \\ 1: & \text{answer}\end{cases}$
(b)	$\frac{dy}{dx}=\frac{-(x+2y)}{(2x+y)}=0 \Rightarrow x=-2y$ $(-2y)^2+4(-2y)y+y^2=-12$ and $-3y^2=-12 \Rightarrow y \pm 2$. Verify that both values of y yield horizontal tangent lines by showing that $\frac{dy}{dx}=0$ in both cases. When $y=2$, $x=-4$ and $\frac{dy}{dx}=\frac{-(-4+4)}{-8+2}=0$. When $y=-2$, $x=4$ and $\frac{dy}{dx}=\frac{-(4-4)}{8-2}=0$.	2: $\begin{cases}1: & \frac{dy}{dx}=0 \\ 1: & \text{solutions}\end{cases}$
(c)	$\left.\frac{dy}{dx}\right\|_{(-4,14)}=\frac{-[-4+2(14)]}{2(-4)+14}=\frac{-24}{6}=-4$ $y-14=-4(x+4) \Rightarrow y=-4x-2$	1: answer
(d)	$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}=\frac{dy/dt}{dx/dt}$ $-4=\frac{-1/2}{dx/dt} \Rightarrow \frac{dx}{dt}=\frac{1}{8}$	2: $\begin{cases}1: & \text{relationship} \\ 1: & \text{answer}\end{cases}$
(e)	$y=14-4(-4.01+4) \Rightarrow y \approx 14.04$	1: answer

4. (a) (*Calculus for AP* 1st ed. pages 173–177; EU 2.1; LO 2.1C; EK 2.1C5; MPAC 2,3,5)
4. (b), (c) (*Calculus for AP* 1st ed. pages 124–130; EU 2.3; LO 2.3B; EK 2.3B1; MPAC 2,3)
4. (d) (*Calculus for AP* 1st ed. pages 189–193; EU 2.1; LO 2.1C; EK 2.1C4; MPAC 2,3,5)
4. (e) (*Calculus for AP* 1st ed. page 267; EU 2.3; LO 2.3B; EK 2.3B2; MPAC 2,3)

5. Let L be the tangent to $f(x)=\sqrt{x}$ at any point on the curve as shown in the diagram to the right.
   (a) Show that the x-intercept of the tangent to the curve at the point $(h, \sqrt{h})$ is $-h$.
   (b) Find the area of the region bounded by the tangent to the curve at $x = 4$, the curve, and the x-axis.

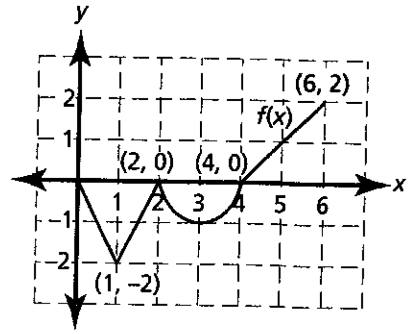


(c) What is the volume when the region found in part b is rotated about the line $x = 4$.

	Solution	Possible points
(a)	$y'(x) = \dfrac{1}{2\sqrt{x}}$ and $y'(h) = \dfrac{1}{2\sqrt{h}}$ L: $y - \sqrt{h} = \dfrac{1}{2\sqrt{h}}(x - h)$ The x-intercept occurs when $y = 0$. Thus $\left(0 - \sqrt{h}\right)2\sqrt{h} = (x - h) \Rightarrow x = -h$.	3: $\begin{cases} 1: & y'(x) \\ 1: & L \\ 1: & y = 0 \to \text{answer} \end{cases}$
(b)	The area is found by integrating with respect to y. Rewriting the functions, L: $y - 2 = \dfrac{1}{4}(x - 4) \Rightarrow x = 4y - 4$ and $x = y^2$ $A = \int_0^2 \left[y^2 - (4y - 4)\right] dy$ $= \left(\dfrac{y^3}{3} - 2y^2 + 4y\right)\Big\|_0^2$ $= \dfrac{8}{3} - 8 + 8 = \dfrac{8}{3}$ The area may also be determined by integrating with respect to x, but two integrals will be needed. L: $y - 2 = \dfrac{1}{4}(x - 4) \Rightarrow y = \dfrac{1}{4}x + 1$, with the x-intercept of -4. $A = \int_{-4}^{0}\left(\dfrac{1}{4}x + 1\right)dx + \int_0^4\left(\dfrac{1}{4}x + 1 - x^{\frac{1}{2}}\right)dx$ $= \left(\dfrac{1}{8}x^2 + x\right)\Big\|_{-4}^{0} + \left(\dfrac{1}{8}x^2 + x - \dfrac{2}{3}x^{\frac{3}{2}}\right)\Big\|_0^4$ $= 0 - (2 - 4) + \left(2 + 4 - \dfrac{16}{3}\right) = \dfrac{8}{3}$	3: $\begin{cases} 1: & \text{integral with limits} \\ 1: & \text{antiderivative} \\ 1: & \text{answer} \end{cases}$
(c)	$V = \pi\int_0^2 \left[4 - (4y - 4)\right]^2 - \left(4 - y^2\right)^2 dy$ $= \pi\int_0^2 16y^2 - \left(16 - 8y^2 + y^4\right)dy$ $= \pi\left(8y^3 - 16y - \dfrac{y^5}{5}\right)\Big\|_0^2 = \dfrac{128\pi}{5}$	3: $\begin{cases} 1: & \text{integral with limits} \\ 1: & \text{antiderivative} \\ 1: & \text{answer} \end{cases}$

5. (a) (*Calculus for AP* 1st ed. pages 134–142; EU 2.3; LO 2.3B; EK 2.3B1; MPAC 2,3,4)
5. (b) (*Calculus for AP* 1st ed. pages 408–413; EU 3.4; LO 3.4D; EK 3.4D1; MPAC 2,3,4)
5. (c) (*Calculus for AP* 1st ed. pages 418–424; EU 3.4; LO 3.4D; EK 3.4D2; MPAC 2,3,4)

6. Let f be a function defined in the closed interval $0 \le x \le 6$. The graph of f consists of three line segments and a semicircle. Let
$g(x) = 3 + \int_2^x f(t)\, dt$.
   (a) Find $g(1)$, $g'(1)$, and $g''(1)$.
   (b) What is the average rate of change of $g(x)$ in the interval $2 \le x \le 6$?
   (c) Identify the x-coordinate of any extrema of $g(x)$ on $0 < x < 6$. Explain your reasoning.
   (d) Identify the x-coordinate of any points of inflection of $g(x)$ on $0 < x < 6$.



	Solution	Possible points
(a)	$g(1) = 3 + \int_2^1 f(t)\, dt = 3 - \int_1^2 f(t)\, dt = 3 + \frac{1}{2}(1)(2) = 4$ $g'(x) = f(x) \Rightarrow g'(1) = -2$ $g''(x) = f'(x) \Rightarrow g''(1)$ does not exist.	3: 1 for each answer
(b)	$\frac{g(6)-g(2)}{6-2} = \frac{\left[3+\left(-\frac{\pi}{2}\right)+\frac{1}{2}(2)(2)\right]-(3)}{4} = \frac{4-\pi}{8}$	2: $\begin{cases}1: \text{difference quotient}\\1: \text{answer}\end{cases}$
(c)	Extrema exist where $g'(x) = f(x) = 0$ or $g'(x)$ does not exist. Extrema exist at $x = 4$ because $g'(4) = f(4)$ is zero and $g'(x)$ (or $f(x)$) changes from negative values to positive values at $x = 4$. There are no sign changes at $x = 2$ so it is not an extremum even though $g'(2) = 0$.	2: $\begin{cases}1 \text{ reason}\\1 \text{ answer}\end{cases}$
(d)	Inflection points occur where $g''(x) = f'(x)$ change sign. The points of inflection exist at $x = 1$, 2, and 3.	2: $\begin{cases}1 \text{ reason}\\1 \text{ answer}\end{cases}$

6. (a) (*Calculus for AP* 1st ed. pages 317–325; EU 2.1,3.3; LO 2.1D,3.3A; EK 2.1D1,3.3A2; MPAC 2,3,4)
6. (b) (*Calculus for AP* 1st ed. pages 220–223; EU 2.1; LO 2.1A; EK 2.1A1; MPAC 2,3,4)
6. (c) (*Calculus for AP* 1st ed. pages 317–325; EU 2.2; LO 2.2A; EK 2.2A1; MPAC 2,3,4,6)
6. (d) (*Calculus for AP* 1st ed. pages 237–241; EU 2.2; LO 2.2A; EK 2.2A1; MPAC 2,3,4)