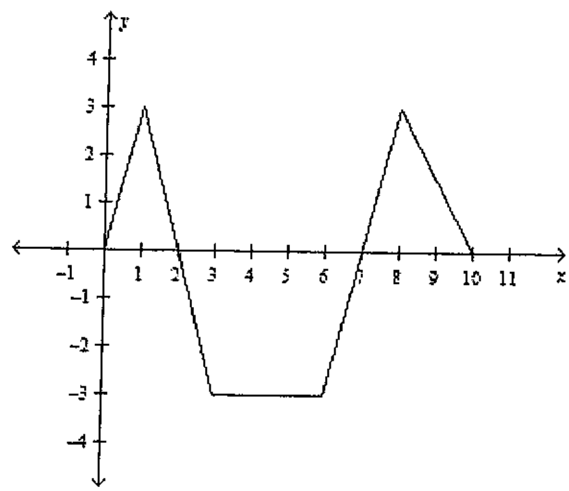
More on Particle Motion
Finding Net and Total Distance

Essential Questions

1) How can the position function be used to determine the distance a particle travels?
2) Compare net distance to total distance. How are they similar? Dissimilar?
3) What role does the velocity function play in determining total distance?

The graph below represents the velocity, $v(t)$ which is measured in meters per second, of a particle moving along the x – axis.



At what value(s) of t does the particle have no acceleration on the interval (0, 10)? Justify your answer.

$a(t)$ DOES NOT EXIST AT $t = 1, 3, 6$ & 8 B/C $v(t)$ HAS CUSPS

Express the acceleration, $a(t)$, as a piecewise-defined function on the interval (0, 10).

$$a(t) = \begin{cases} 3, & 0 < t < 1 \\ -3, & 1 < t < 3 \\ 0, & 3 < t < 6 \\ 3, & 6 < t < 8 \\ -\frac{3}{2}, & 8 < t < 10 \end{cases}$$

For what value(s) of t is the particle moving to the right? To the left? Justify your answer.

PARTICLE IS MOVING RIGHT ON $(0, 2) \cup (7, 10)$ B/C $v(t) > 0$

PARTICLE IS MOVING LEFT ON $(2, 7)$ B/C $v(t) < 0$

Find the average acceleration of the particle on the interval [1, 8]. Show your work.

$$\text{AVERAGE ACCELERATION} = \frac{v(1) - v(8)}{1 - 8} = \frac{3 - 3}{-7} = 0 \text{ METERS/SEC}^2$$

Definition of Net Distance:	THE DISTANCE BETWEEN THE POINT OF ORIGIN AND THE FINAL POSITION
Definition of Total Distance:	THE SUM OF ALL DISTANCES MOVED IN ANY DIRECTION

If a particle is moving in the same direction the entire amount of time, what can be said about the net distance and the total distance?

IF THE PARTICLE NEVER CHANGES DIRECTIONS,

NET DISTANCE = TOTAL DISTANCE

To Find the Net Distance a Particle Travels on an Interval $[a, b]$

$$|p(a) - p(b)|$$

To Find the Total Distance a Particle Travels on an Interval

IF THE PARTICLE CHANGES DIRECTION @ $t = c$

$$|p(a) - p(c)| + |p(c) - p(b)|$$

The position of a particle is given by the function $p(t) = 2t^3 - 6t^2 + 8t$ where $p(t)$ is measured in centimeters. Find the net and total distance the particle travels from $t = 1.5$ seconds to $t = 4$ seconds.

NET DISTANCE = $|p(1.5) - p(4)| = 58.75$ cm

SINCE $p(t)$ IS ALWAYS INCREASING ON $(1.5, 4)$, $v(t)$ IS ALWAYS POSITIVE, AND THE PARTICLE NEVER CHANGES DIRECTION

$\therefore$ NET DISTANCE = TOTAL DISTANCE = 58.75 cm

The position of a particle is given by the function $p(t) = e^{2t} - 8t$ where $p(t)$ is measured in feet. Find the net and total distance the particle travels from $t = 0.5$ minutes to $t = 1.5$ minutes.

$$N.D. = |p(0.5) - p(1.5)| = 9.367 \text{ FT.}$$

$c = 0.69314828$

$$T.D. = |p(0.5) - p(c)| + |p(c) - p(1.5)| = 9.894 \text{ FT}$$

The position of a particle is given by the function $p(t) = t + 2\sin t$ where $p(t)$ is measured in feet. Find the net and total distance the particle travels from $t = \frac{\pi}{6}$ minutes to $t = \frac{5\pi}{4}$ minutes.

$$N.D. = \left|p\left(\frac{\pi}{6}\right) - p\left(\frac{5\pi}{4}\right)\right| = 0.989 \text{ FT}$$

$c = 2.0943945$

$$T.D. = \left|p\left(\frac{\pi}{6}\right) - p(c)\right| + \left|p(c) - p\left(\frac{5\pi}{4}\right)\right| = 3.617 \text{ FT.}$$

NO CALCULATOR PERMITTED

A particle moves along the x – axis so that its position at any time $t \geq 0$ is given by the function $p(t) = t^3 - 4t^2 - 3t + 1$, where p is measured in feet and t is measured in seconds.

a. Find the average velocity on the interval $t = 1$ and $t = 2$ seconds. Give your answer using correct units.

b. On what interval(s) of time is the particle moving to the left? Justify your answer.

c. Using appropriate units, find the value of $p'(3)$ and $p''(3)$. Based on these values, describe the motion of the particle at $t = 3$ seconds. Give a reason for your answer.

d. What is the maximum velocity on the interval from $t = 1$ to $t = 3$ seconds. Show the analysis that leads to your conclusion.

e. Find the total distance that the particle moves on the interval [1, 5]. Show and explain your analysis.

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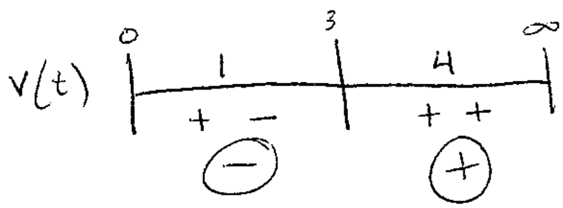
a) AVG. VELOCITY $= \dfrac{p(1) - p(2)}{1 - 2} = \dfrac{-5 - (-13)}{-1} = -8$ FT/SEC

b) $v(t) = 3t^2 - 8t - 3$

$(3t + 1)(t - 3) = 0$

$t = -\frac{1}{3}, 3$



PARTICLE IS MOVING LEFT ON $(0, 3)$ B/C $v(t) < 0$

c) $p'(t) = 3t^2 - 8t - 3$

$p'(3) = 3(3)^2 - 8(3) - 3$

$p'(3) = 0$

$p''(t) = 6t - 8$

$p''(3) = 6(3) - 8$

$p''(3) = 10$

SINCE VELOCITY IS ZERO, BUT ACCELERATION IS NOT ZERO, THE PARTICLE IS MOMENTARILY STOPPED AND CHANGING DIRECTIONS

d) $v(t) = 3t^2 - 8t - 3$

$v(1) = -8$ FT/SEC

$v\left(\frac{4}{3}\right) = -\frac{25}{3}$ FT/SEC

$v(3) = 0$ FT/SEC

$v'(t) = 6t - 8$

$v'(t) = 6t - 8 = 0$

$t = \frac{4}{3}$

MAX VELOCITY IS 0 FT/SEC

e) $|p(1) - p(3)| + |p(3) - p(5)| = |-5 - (-17)| + |-17 - 11| = 40$ FT.

CALCULATOR PERMITTED

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table below

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per min)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

a. Find the average acceleration on the interval $5 \leq t \leq 20$. Express your answer using correct units of measure.

b. Based on the values in the table, on what interval(s) is the acceleration of the plane guaranteed to equal zero on the open interval $0 < t < 40$? Justify your answer.

c. Does the data represent velocity values of the plane moving away from its point of origin or returning to its point of origin? Give a reason for your answer.

d. The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? What does this value indicate about the velocity at $t = 23$? Justify your answer, indicating units of measure.

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a) AVERAGE ACCELERATION ON $[5, 20] = \dfrac{v(5) - v(20)}{5 - 20} = \dfrac{9.2 - 4.5}{-15} = 0.313$ MILES/MIN^2

b) SINCE $v(t)$ IS CONTINUOUS ON $[25, 30]$, DIFFERENTIABLE ON $(25, 30)$, AND $v(25) = v(30) = 2.4$, ROLLE'S THEOREM GUARANTEES A VALUE $t = c$ SUCH THAT $a(c) = 0$ ON $(25, 30)$.

c) SINCE $v(t)$ IS ALWAYS POSITIVE, THE PARTICLE IS ALWAYS MOVING RIGHT AND NEVER CHANGES DIRECTIONS. THEREFORE, THE PARTICLE IS ALWAYS MOVING AWAY FROM ITS POINT OF ORIGIN.

d) $f'(t) = a(t) = -\frac{1}{10}\sin\left(\frac{t}{10}\right) + \frac{21}{40}\cos\left(\frac{7t}{40}\right)$

$a(23) = -\frac{1}{10}\sin\left(\frac{23}{10}\right) + \frac{21}{40}\cos\left(\frac{7(23)}{40}\right) = -0.408$ MILES/MIN^2

SINCE $a(23) < 0$, THE VELOCITY IS DECREASING AT $t = 23$

You must show your work to earn credit for the following. You will need to use a calculator for these.

If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c on the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following values could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

AROC

$$\frac{f\left(\frac{\pi}{2}\right) - f\left(\frac{3\pi}{2}\right)}{\frac{\pi}{2} - \frac{3\pi}{2}}$$

$$\frac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}}{-\pi}$$

$$0$$

IROC

$$f'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right)$$

$$\frac{1}{2}\cos\left(\frac{x}{2}\right) = 0$$

$$\cos\left(\frac{x}{2}\right) = 0$$

$$\frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \pi, \cancel{3\pi}$$

NOT ON INTERVAL

A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

(A) -5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

$$v(t) = 4\sin t - t$$

$$a(t) = 4\cos t - 1 = 0$$

$$\cos t = \frac{1}{4}$$

$$t = \cos^{-1}\left(\frac{1}{4}\right)$$

$$t = 1.318$$

$$v(1.318) = 4\sin(1.318) - 1.318 = 2.55$$