Name_____ Date_____ Class_____

Homework 5.4

A particle moves along the x axis such that its position, for $t > 0$, is given by the function $p(t) = e^{2t} - 5t$. Use this information to complete exercises 1 – 4.

1. What are the values of $p'(2)$ and $p''(2)$? Explain what each value represents.

$p'(t) = 2e^{2t} - 5$

$p'(2) = 2e^{2(2)} - 5$

$\boxed{p'(2) = 2e^4 - 5}$

$p''(t) = 4e^{2t}$

$p''(2) = 4e^{2(2)}$

$\boxed{p''(2) = 4e^4}$

$p'(2)$ REPRESENTS THE VELOCITY OF THE PARTICLE @ $t = 2$

$p''(2)$ REPRESENTS THE ACCELERATION OF THE PARTICLE @ $t = 2$

2. Based on the values found in part (a), what can be concluded about the speed of the particle at $t = 2$? Give a reason for your answer.

SINCE $p'(2) > 0$ AND $p''(2) > 0$, THE SPEED OF THE PARTICLE IS INCREASING @ $t = 2$

3. On what interval(s) of t is the particle moving to the left? To the right? Justify your answers.

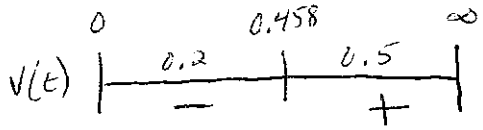
$p'(t) = v(t) = 2e^{2t} - 5 = 0$

$e^{2t} = \frac{5}{2}$

$2t = \ln \frac{5}{2}$

$t = \frac{1}{2} \ln \frac{5}{2}$

$t = 0.458145$



LEFT ON $\left(0, \frac{1}{2} \ln \frac{5}{2}\right)$ B/C $v(t) < 0$

RIGHT ON $\left(\frac{1}{2} \ln \frac{5}{2}, \infty\right)$ B/C $v(t) > 0$

4. Does the particle ever change directions? Justify your answer.

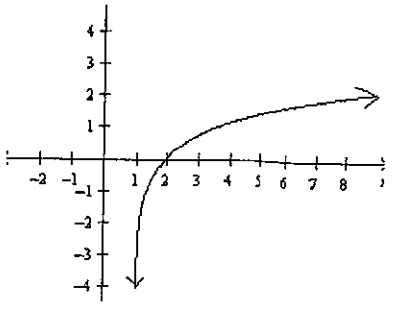
YES @ $t = 0.458145$ $v(t)$ CHANGES SIGNS

5. The graph of $v(t)$, the velocity of a moving particle, is given below. What conclusions can be made about the movement of the particle along the x – axis and the acceleration, $a(t)$, of the particle for $t > 0$? Give reasons for your answers.

SINCE $v(t) < 0$ ON $(0, 2)$, PARTICLE IS MOVING LEFT

SINCE $v(t) > 0$ ON $(2, \infty)$, PARTICLE IS MOVING RIGHT

SINCE $v(t)$ IS ALWAYS INCREASING, ACCELERATION IS ALWAYS POSITIVE



6. If the position of a particle is defined by the function $x(t) = t^3 - 9t^2 + 24t$ for $t > 0$, is the speed of the particle increasing or decreasing when $t = 2.5$? Justify your answer.

$v(t) = 3t^2 - 18t + 24$

$v(t) = 3(t^2 - 6t + 8)$

$v(t) = 3(t-4)(t-2)$

$v(2.5) = 3(-1.5)(0.5) < 0$

$a(t) = 6t - 18$

$a(t) = 6(t-3)$

$a(2.5) = 6(0.5) > 0$

THE SPEED OF THE PARTICLE IS DECREASING @ $t = 2.5$ B/C $v(2.5) < 0$ AND $a(2.5) > 0$

The position of a particle is given by the function $p(t) = (2t-3)e^{2-t}$ for $t > 0$. Answer questions 7 – 9.

7. What is the average velocity from $t = 1$ to $t = 3$?

$$\text{AVG. VELOCITY} = \frac{p(1) - p(3)}{1 - 3} = \frac{-2.718282 - 1.1036383}{-2} = 1.911$$

8. Find an equation for $v(t)$, the velocity of the particle.

$v(t) = (2t-3) \cdot e^{2-t} \cdot (-1) + 2e^{2-t}$

$v(t) = -(2t-3)e^{2-t} + 2e^{2-t}$

9. For what value(s) of t will the $v(t) = 0$?

$v(t) = 0$

@ $t = 2.5$

2003 AP Calculus AB #2 (Partial)

A particle moves along the $x-$axis so that its velocity at time t is given by $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$.

10. Find the acceleration of the particle at $t = 2$. Is the speed of the particle increasing at $t = 2$? Explain why or why not.

$a(2) = 1.588$

$v(2) = -2.728$

THE SPEED OF THE PARTICLE IS DECREASING B/C $a(2)$ AND $v(2)$ HAVE DIFFERENT SIGNS

11. Find all times in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

PARTICLE CHANGES DIRECTION @ $t = 2.507$ B/C $v(t)$ CHANGES FROM NEGATIVE TO POSITIVE