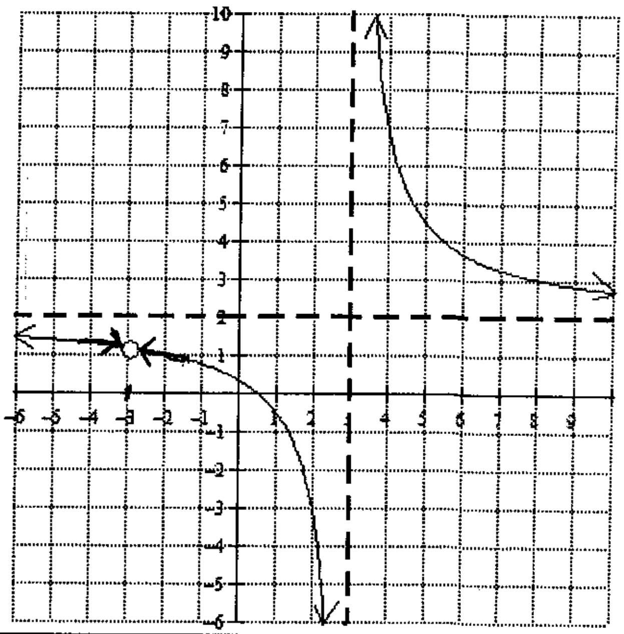
Understanding the Limit
A Numerical and Graphical Approach

The equation of the function graphed to the right is $f(x) = \frac{2x^2+5x-3}{x^2-9}$. The coordinates of the hole in the graph are $\left(-3, \frac{7}{6}\right)$.



Pre-calculus Statements	Calculus Limit Notation
As $x \to -\infty$, the graph of $f(x) \to$ <u>2</u>.	$\lim_{x \to -\infty} f(x) = 2$
As $x \to \infty$, the graph of $f(x) \to$ <u>2</u>.	$\lim_{x \to \infty} f(x) = 2$
As $x \to -3$ from the left, the graph of $f(x) \to$ $\frac{7}{6}$.	$\lim_{x \to -3^-} f(x) = \frac{7}{6}$
As $x \to -3$ from the right, the graph of $f(x) \to$ $\frac{7}{6}$.	$\lim_{x \to -3^+} f(x) = \frac{7}{6}$
As $x \to 3$ from the left, the graph of $f(x) \to$ $-\infty$.	$\lim_{x \to 3^-} f(x) = -\infty$
As $x \to 3$ from the right, the graph of $f(x) \to$ ∞.	$\lim_{x \to 3^+} f(x) = \infty$

Based on what you have just seen, how might you informally define what the value of a limit represents in terms of the graph?

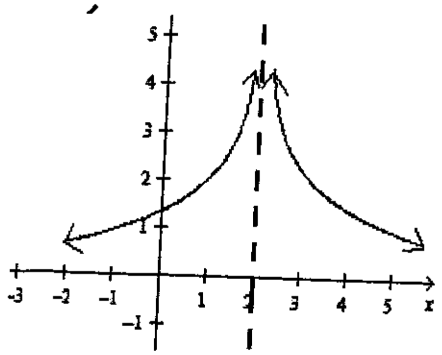
$\lim_{x \to a} f(x)$ is the y value the graph approaches as $x \to a$

Limit Existence Theorem

$\lim_{x \to a} f(x)$ exists if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = b$, where b is any REAL NUMBER.

Limits That Do Not Exist

Example #1



Find each of the following from the graph.

a) $\lim_{x \to 2^-} f(x) = \infty$

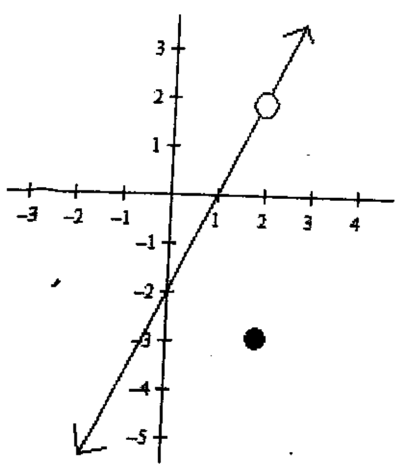
b) $\lim_{x \to 2^+} f(x) = \infty$

c) $f(2) =$ UNDEFINED

d) Does $\lim_{x \to 2} f(x)$ exist or not? Why or why not?

NO, $\lim_{x \to 2^-} f(x) = \infty$ & ∞ IS NOT A REAL NUMBER

Example #2



Find each of the following from the graph.

a) $\lim_{x \to 2^-} f(x) = 2$

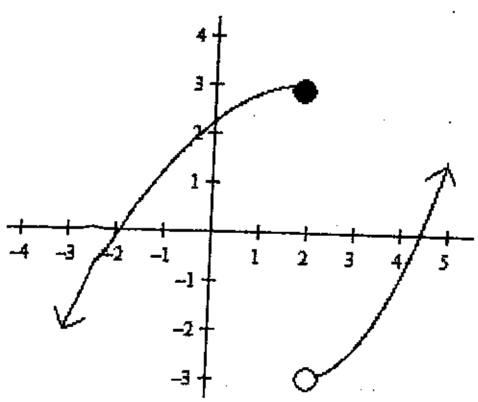
b) $\lim_{x \to 2^+} f(x) = 2$

c) $f(2) = -3$

d) Does $\lim_{x \to 2} f(x)$ exist or not? Why or why not?

YES B/C $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 2$

Example #3



Find each of the following from the graph.

a) $\lim_{x \to 2^-} f(x) = 3$

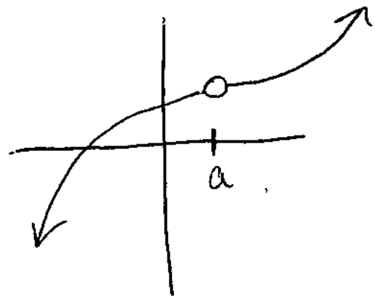
b) $\lim_{x \to 2^+} f(x) = -3$

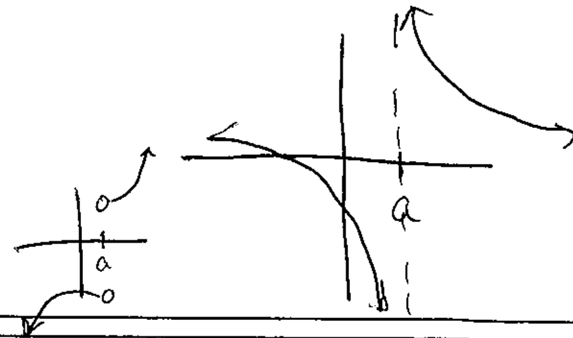
c) $f(2) = 3$

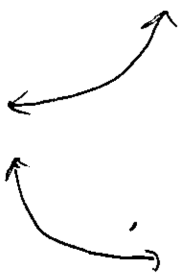
d) Does $\lim_{x \to 2} f(x)$ exist or not? Why or why not?

NO, B/C $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

Based on what you have seen so far, does $f(a)$ have to be defined in order for the $\lim_{x \to a} f(x)$ to exist? Draw and explain two different graphs to justify your reasoning. In both graphs, $f(a)$ should be undefined but in one graph, the limit should exist while in the second one, it should not exist.







Below is a table of values of an exponential function. Use the table to find the limits that follow.

x	−9	−5	−3	−1	1	3	9
$f(x)$	513	33	9	3	1.5	1.125	1.002

a) $\lim_{x \to -\infty} f(x) = \infty$

b) $\lim_{x \to -3} f(x) = 9$

c) $\lim_{x \to 1} f(x) = 1.5$

d) $\lim_{x \to \infty} f(x) = 1$

Below is a table of values of a rational function. Use the table to find the limits that follow.

x	−1000	−1.001	−1	−0.999	0	1.999	2	2.001	1000
$F(x)$	1.002	2001	Undefined	−1999	−1	0.333	Undefined	0.334	0.998

a) $\lim_{x \to -\infty} f(x) = 1$

b) $\lim_{x \to -1^-} f(x) = \infty$

c) $\lim_{x \to -1^+} f(x) = -\infty$

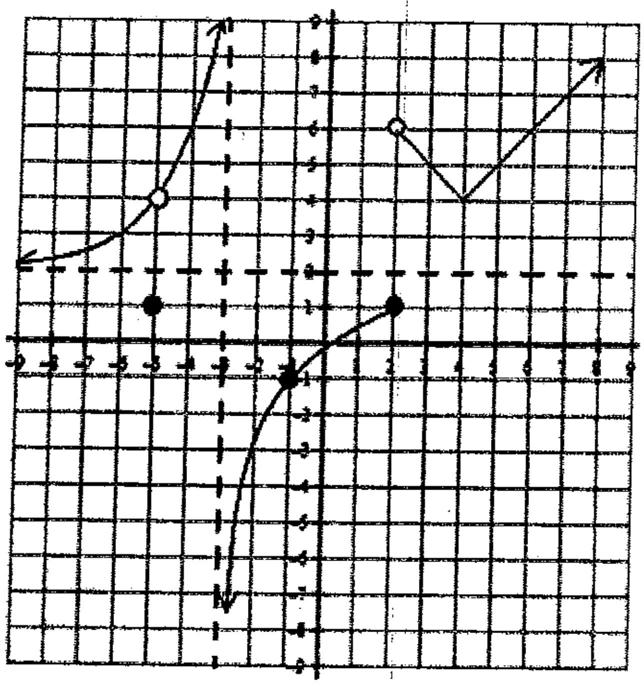
d) $\lim_{x \to 2} f(x) = \frac{1}{3}$

e) $\lim_{x \to \infty} f(x) = 1$

f) $\lim_{x \to -1} f(x)$ DNE

A Graphical Analysis of Limits

- Consider the graph of the function, f(x), graphed below.



A Graphical Analysis of Limits

Using the graph, find the value of each of the following limits. If a limit does not exist, state why.

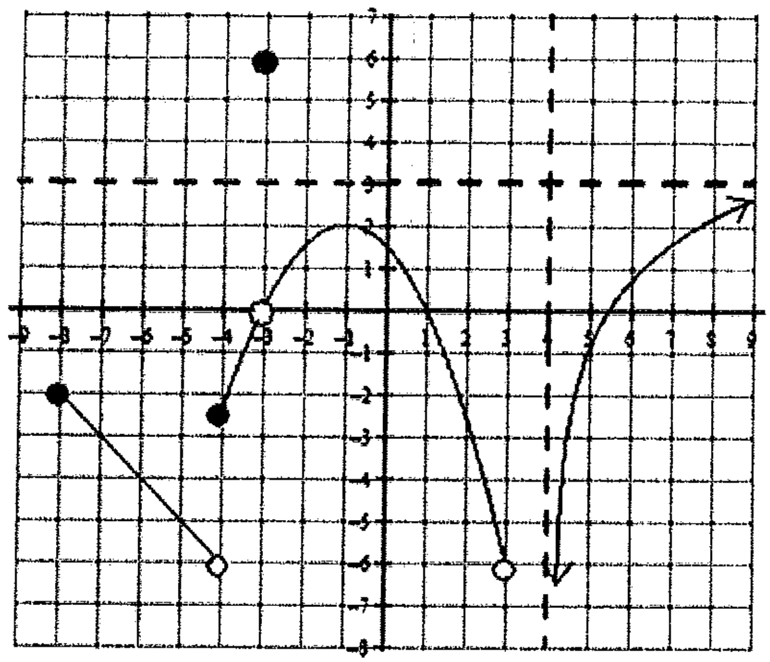
a) $\lim_{x \to -3^-} f(x) = \infty$ b) $\lim_{x \to -5} f(x) = 4$ c) $\lim_{x \to -1} f(x) = -1$

c) $\lim_{x \to -3} f(x)$ DNE d) $\lim_{x \to 2^-} f(x) = 1$ e) $\lim_{x \to 2^+} f(x) = 6$

f) $\lim_{x \to 2} f(x)$ DNE g) $\lim_{x \to -\infty} f(x) = 2$ h) $\lim_{x \to \infty} f(x) = \infty$

A Graphical Analysis of Limits

Consider the graph of the function, g(x), graphed below.



A Graphical Analysis of Limits

Find the value of each of the following limits using the graph of g(x). If a limit does not exist, state why.

a) $\lim_{x\to -3^-} g(x) = 0$

b) $\lim_{x\to -6} g(x) = -4$

c) $\lim_{x\to -1^+} g(x) = 2$

d) $\lim_{x\to -3^+} g(x) = 0$

e) $\lim_{x\to 4^-} g(x) =$ DNE

f) $\lim_{x\to 4^+} g(x) = -\infty$

g) $\lim_{x\to 4} g(x)$ DNE

h) $\lim_{x\to -4^+} g(x) = -2.5$

i) $\lim_{x\to -4^-} g(x) = -6$